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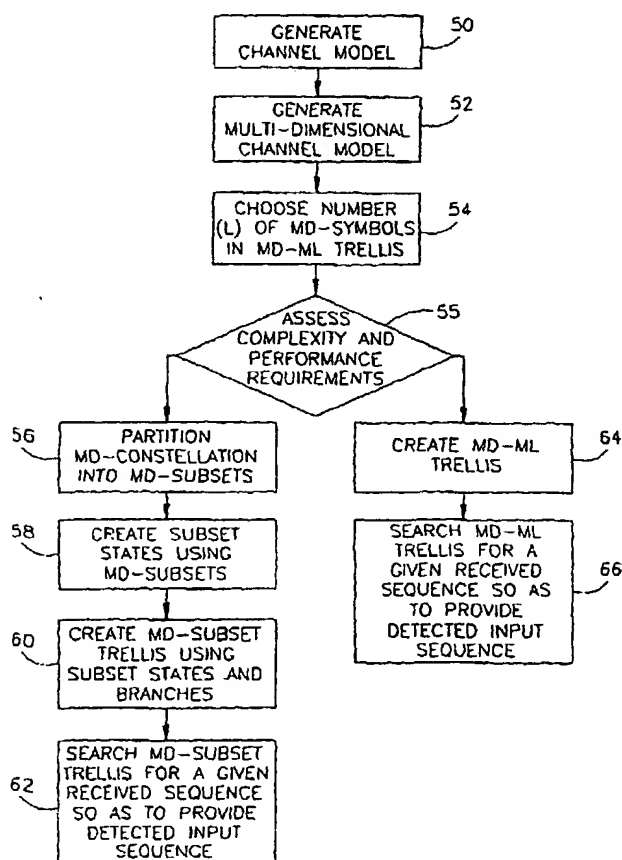
(57) Abstract: A method of detecting symbols in a signal transfer system, comprising: transferring symbols on a channel which may distort the symbols; representing the transferred symbols by multi-dimensional (MD) symbols, which each includes a plurality of transferred symbols; providing a trellis which has states which are defined by one or more MD symbols, and which has branches which correspond to respective MD symbols; receiving a plurality of samples; and calculating estimate values of one or more transferred symbols based on the received samples and the provided trellis.

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REDUCED STATE SEQUENCE ESTIMATION SIGNAL DETECTION METHOD

FIELD OF THE INVENTION

The present invention relates to signal detection and in particular to detection of signals using trellis systems.

BACKGROUND OF THE INVENTION

Communication channels generally carry information in the form of symbols which represent one or more binary bits. Communication channels generally add noise to the symbols transmitted on them. In addition, communication channels smear out the signals transmitted on them, in a process referred to as inter-symbol interference (ISI). Receivers generally include an equalizer that determines the transmitted symbols based on the symbols received from the channel.

Fig. 1 is a schematic block diagram of a transmission system 20. A transmitter 22 transmits a sequence of symbols $\{X_n\}$ over a channel 24, to a receiver 26. Each symbol X_n may take on one value from a set of values referred to as an alphabet or constellation. The number of values in the constellation is referred to as D. Channel 24 distorts the sequence of symbols $\{X_n\}$ in a manner which is generally modeled by a convolution with a function $\{f_k\}$ having $K+1$ non-zero terms, such that receiver 26 receives symbols $\{r_n\}$ which have the modeled form of:

$$r_n = \sum_{k=0}^{K} f_k x_{n-k} + w_n$$

in which w_n represents noise added by channel 24. The number of terms in function $\{f_k\}$ is $(K+1)$ and is referred to as the length of channel 24. An equalizer 28 generates respective estimates $\{\tilde{x}_n\}$ (referred to also as detections) of the transmitted symbols $\{X_n\}$ from the received symbols $\{r_n\}$, based on a model $\{f_k\}$ of channel 24.

There exist various types of equalizers which perform their task (referred to as equalization), using different methods. These methods include linear equalization (LE), decision-feedback equalization (DFE) and maximum likelihood sequence estimation (MLSE). The MLSE method chooses from among a set of possible transmitted sequences a single sequence which is most likely to have been transmitted in view of the received symbols and a model of the channel. In a typical MLSE application, a state machine (referred to as a trellis) is created in which each possible transmitted sequence is represented by a path going through a plurality of states. Typically, each state represents a short sequence of a predetermined number

of recently transmitted symbols. Each transition between states (referred to as a branch of the trellis) represents a single transmitted symbol. While MLSE has much lower error rates than the other methods, its computation and space complexity is enormous and therefore in many cases its use is substantially impossible.

5   A document titled "Reduced-State Sequence Estimation with Set Partitioning and Decision Feedback", M.V. Eyuboglu, S.U.H. Qureshi, IEEE Trans. Commun., Vol. 36, No. 1, January 1988, suggests a simplification of MLSE referred to as reduced state sequence estimation (RSSE). In the RSSE method, set partitioning methods are used in order to reduce the number of states in the trellis. The RSSE method introduces a subset trellis with subset

10   states, in which each subset state represents one or more states of the MLSE.

SUMMARY OF THE INVENTION

An aspect of some embodiments of the present invention relates to an equalizer which performs symbol sequence estimation using a trellis in which states and transitions are defined based on a multi dimensional (MD) system model. In the MD model, a plurality of transmitted

15   symbols are viewed as a single MD-symbol. Particularly, in some embodiments of the invention, the branches of the MD subset trellis represent one or more multi-dimensional symbols which are formed of a plurality of transmitted symbols. Using the MD model, set partitioning methods may be used on the MD-constellation to generate an MD-subset trellis which has lower computation and/or space complexity than can be achieved using the prior art

20   for predetermined performance requirements. Alternatively, better performance than prior art methods may be achieved with comparable computation and/or space complexity.

In some embodiments of the invention, the states of the trellis comprise subsets which represent a plurality of MD-symbol value permutations.

In some embodiments of the invention, an MD-ML (maximal likelihood) trellis is

25   defined based on the MD model. The MDML trellis is searched to provide a detected symbol sequence for a received sample sequence. These embodiments are referred to as MD-MLSE (multi dimensional - maximal likelihood sequence estimator).

In some embodiments of the invention, an MD-subset trellis is defined based on the MD model and a partitioning of the MD constellation of the model into MD subsets. The MD-

30   subset trellis is searched to provide a detected symbol sequence for a received sample sequence. These embodiments are referred to as MD-RSSE (Multi-dimensional - reduced state sequence estimator). In some embodiments of the invention, an MD-RSSE trellis is

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constructed in a manner which optimizes a tradeoff, for example, between performance and complexity.

In some embodiments of the invention, the transmission is indifferent to the MD system model of the equalizer. Alternatively, the transmitter optimizes the transmitted sequence based on knowledge of the structure of the MD-RSSE or other decoding method which is used by a receiving end.

In some embodiments of the invention, the performance and/or complexity of the MD-RSSE are analyzed and accordingly the method is tailored to a specific channel and/or to specific performance requirements.

A potential advantage of some embodiments of the invention is that the equalization method can be analyzed.

In some embodiments of the invention, the sequence estimation methods described herein are used as a cost calculation method for selecting an optimal transmission method. In one example, a transmission sequence is selected from among a plurality of transmission sequences, using an MD-RSSE, instead of, for example, a reduced Viterbi method, as used in the art. Substantially any desired cost function may be used, for example a function modeling expected channel noise statistics. It should be noted that the receiver may be any receiver and does not necessarily use the equalizer methods described herein.

There is thus provided in accordance with an exemplary embodiment of the invention, a method of detecting symbols in a signal transfer system, comprising:

transferring symbols on a channel which may distort the symbols;

representing the transferred symbols by multi-dimensional (MD) symbols, which each includes a plurality of transferred symbols;

providing a trellis which has states which are defined by one or more MD symbols, and which has branches which correspond to respective MD symbols;

receiving a plurality of samples; and

calculating estimate values of one or more transferred symbols based on the received samples and the provided trellis.

Optionally, the MD symbols comprise a plurality of transferred symbols which take on values from identical value sets. Alternatively or additionally, at least one of the states of the provided trellis comprises a subset state which represents a plurality of different MD-symbol values. Optionally, the trellis comprises states which represent values of a predetermined

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number of MD symbols and wherein providing the trellis comprises generating the trellis by partitioning the possible values of each of the MD symbols in the states of the trellis into one or more subsets and wherein the states of the trellis comprise all the permutations of the subsets of the values of the multi-dimensional symbols. Optionally, partitioning the possible

5   values of each of the MD symbols in the states of the trellis into one or more subsets comprises grouping the possible values of different MD symbols of the trellis states into different numbers of subsets. Alternatively, partitioning the possible values of each of the MD symbols in the states of the trellis into one or more subsets comprises partitioning the possible values of the MD symbols which include earlier transmitted symbols into no more subsets than the

10   possible values of the MD symbols which were transmitted later were partitioned into.

In an exemplary embodiment of the invention, partitioning the possible values of each of the MD symbols in the states of the trellis into one or more subsets comprises partitioning the possible values of later transmitted symbols into subsets of which all the members are included in a single subset of earlier transmitted symbols.

15   In an exemplary embodiment of the invention, the method comprises generating a model of the channel and wherein providing the trellis comprises providing a trellis generated responsive to the channel model. Alternatively or additionally, wherein providing the trellis comprises providing a predetermined trellis. Alternatively or additionally, providing the trellis comprises providing a trellis determined responsive to the received samples. Alternatively or

20   additionally providing the trellis comprises optimizing a trellis for a particular transmission situation. Alternatively or additionally providing the trellis comprises:

generating a plurality of possible trellises;

determining for each of the possible trellises one or more of a performance or a complexity parameter; and

25   selecting one of said trellises responsive to said determining.

In an exemplary embodiment of the invention, an equalizer, comprises:

a input channel for receiving received signals;

a processor for applying a MD trellis methods as described herein; and

an output channel for providing an estimate of symbols received on said input channel,

30   based on said applying. Optionally, the equalizer is incorporated in a cellular telephone.

BRIEF DESCRIPTION OF FIGURES

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Particular non-limiting embodiments of the invention will be described with reference to the following description of embodiments in conjunction with the figures. Identical structures, elements or parts which appear in more than one figure are preferably labeled with a same or similar number in all the figures in which they appear, in which:

Fig. 1 is a schematic block diagram of a transmission system known in the art;

Fig. 2 is a flowchart illustrating a method for producing a trellis for symbol decoding, in accordance with an exemplary embodiment of the present invention;

Fig. 3 illustrates a multi-dimensional view of a communication system, in accordance with an embodiment of the present invention;

Fig. 4 is a schematic illustration of producing a trellis for an exemplary channel, in accordance with an exemplary embodiment of the present invention;

Fig. 5 is a schematic block diagram of a data structure managed by an equalizer, in accordance with an exemplary embodiment of the present invention;

Fig. 6 is a flowchart of the acts performed by an equalizer in decoding received signals, in accordance with an embodiment of the present invention;

Fig. 7 is a schematic block diagram of a multi-input multi-output (MIMO) symbol detector, known in the art;

Fig. 8 is a schematic map of a four-way set partitioning for the rectangular 2D 16 QAM signal set, in accordance with an embodiment of the present invention; and

Fig. 9 is a schematic illustration of four possible set partitionings for various channels, in accordance with an exemplary embodiment of the present invention.

DETAILED DESCRIPTION OF EMBODIMENTS

Fig. 2 is a flowchart illustrating a method for producing a trellis for symbol detection, in accordance with an exemplary embodiment of the present invention. Given a communication channel 24, a one dimensional model of the channel is generated (50) using any method known in the art, for example by transmitting a known test sequence through the channel 24 and determining the effect of the channel on the test sequence. In some embodiments of the invention, the model is the form of an FIR (finite impulse response) filter function, with K+1 taps (i.e., coefficients).

For the following description an exemplary channel modeled by the following function:

$$f_0 = 1$$
$$f = [f_1, f_2, f_3, f_4] = [0.1, -0.1, 0.2, 0.8] \tag{1}$$

and having a constellation (alphabet) of two (D=2) possible symbol values {-1,+1}, is used.

Based on the channel model, a number of symbols (K) to be included in states (denoted p_n) of the trellis, is chosen. Optionally, the chosen number of symbols (K) is equal to the length of the channel, i.e., the number of non-negligible coefficients in the model of the
5   channel (excluding f_0). In the example of equation (1), the number of symbols (K) included in states p_n of the trellis is K=4.

Alternatively to generating the model and selecting K based on the model, an estimate of the length of the model is used. Thus, the trellis may be generated before a model of the channel is generated. This may be used in cases in which the channel changes over time.

10   - At any time n, the K symbols recently transmitted by transmitter 22 may take on any of the permutations $p_n = [x_{n-1}, x_{n-2}, \ldots, x_{n-K}]$ of length K of the symbols in the constellation of channel 24. Permutations p_n are referred to herein as trellis states. In the MLSE method known in the art, a maximum likelihood (ML) trellis is built, based on the states p_n. The states p_n and transitions x_n uniquely define the next state p_{n+1}. In the RSSE method known in the art, a
15   subset trellis is built, based on subset-states t_n, which together with transitions x_n uniquely define the next subset-state t_{n+1}. The subset-state t_n of the subset-trellis is equal to a set of states of the ML-trellis.

In some embodiments of the invention, a multi-dimensional (MD) model for system 20 is created (52), as shown in Fig. 3 which illustrates a multi-dimensional view of a
20   communication system 30, in accordance with an embodiment of the present invention. The creation of the MD model may be explicit or implicit. In a multi-dimensional approach to system 20, although transmitter 22 transmits a sequence of single dimension symbols {x_n} over channel 24, to receiver 26, the transmitter is viewed as transmitting for MD-symbols { $\underline{x}_n$ }. Each MD-symbol $\underline{x}_n$ takes the following form:

$$\underline{x}_n = \begin{bmatrix} x_{nM+M-1} \\ x_{nM+M-2} \\ \vdots \\ x_{nM+1} \\ x_{nM} \end{bmatrix}$$

25
in relation to the symbols X_n which were actually transmitted.

6

Each multi-dimensional symbol $\underline{x}_n$ may take on one value from a set of values referred to as an MD-alphabet or MD-constellation. The number of values in the constellation is equal to D^M, as each of the M symbols of the MD-symbols may take on any one of D possible values.

5   In some embodiments of the invention, a number of MD symbols (L) to be included in the states of the created trellis is chosen (54). In some embodiments of the invention, $L = \left\lceil \dfrac{K}{M} \right\rceil$. Optionally, LM = K, when K is divisible by M. Alternatively, when K is not divisible by M, one or more padding 0 taps are concatenated to the filter.

In some embodiments of the invention, a multi-dimension distortion model of channel
10   24 is created as a set of L+1 matrices $\{F_k\}$, which take on the form:

$$
F_k = \begin{bmatrix}
f_{kM} & f_{kM+1} & \cdots & f_{kM+M-1} \\
f_{kM-1} & f_{kM} & \cdots & f_{kM+M-2} \\
\vdots & \vdots & & \vdots \\
f_{kM-M+1} & f_{kM-M+2} & \cdots & f_{kM}
\end{bmatrix}
$$

where f_k are the taps of the one dimensional channel model.

For the simplicity of the following description, the received output samples are viewed as MD-output-samples, in which each MD output symbol $\underline{r}_n$ includes a plurality of received symbols r_n. In an exemplary embodiment of the invention, the following equation relates
15   between the MD-output-samples and the one-dimensional output samples r_n:

$$
\underline{r}_n = \begin{bmatrix}
r_{nM+M-1} \\
r_{nM+M-2} \\
\vdots \\
r_{nM+1} \\
r_{nM}
\end{bmatrix}
$$

Thus, the relation between the transmitted MD-input symbols $\underline{x}_n$ and the MD-received output samples $\underline{r}_n$ is modeled as:

$$
\begin{aligned}
\underline{r}_n &= \sum_{k=0}^{L} F_k \underline{x}_{n-k} + \underline{w}_n \\
&= \underline{y}_n + \underline{w}_n
\end{aligned}
$$

where $\underline{w}_n$ is a proper noise vector.

Optionally, the complexity and/or performance requirements of the equalizer are assessed (55) and accordingly a trellis type is chosen.

In some embodiments of the invention, for example when a high performance is
5   required, a multi-dimensional maximal likelihood (MD-ML) trellis is constructed (64). The MD-ML trellis states p_n are defined by the last L MD-symbols transmitted by the transmitter 22:

$$p_n = [\underline{x}_{n-1}, \underline{x}_{n-2}, \cdots, \underline{x}_{n-L}]$$.

The MD-ML trellis states, together with the currently transmitted MD-symbol $\underline{x}_n$, (which
10   defines an MD-ML trellis transition) uniquely define the next MD-ML state, p_{n+1}. Each transition of the MD-ML trellis is equal to M consecutive transitions in one-dimensional ML-trellises known in the art, such as described in G.D. Forney, Jr., "Lower Bounds on Error Probability in the Presence of Large Intersymbol Interference", IEEE Trans. Commun., February 1972, the disclosure of which is incorporated herein by reference. This trellis can be
15   searched (66), for example, using a Viterbi algorithm, in order to detect the transmitted MD-symbols $\underline{x}_n$ based on the received MD symbols $\underline{r}_n$. Generally, the searching of the MD-ML trellis achieves optimal performance based on the maximum likelihood criteria but in some cases requires high computation and/or complexity.

Alternatively or additionally, an MD-subset trellis is constructed. The MD-subset trellis
20   comprises subset-states t_n which are defined by grouping together a subset of one or more states p_n of the MD-ML trellis. In some embodiments of the invention, the MD constellation of each MD symbol $\underline{x}_{n-k}$ (k=1..L) in state p_n is partitioned (56) into an M-dimensional subset, i.e., all the possible values of each MD symbol $\underline{x}_{n-k}$ are partitioned into one or more subsets. The partitioning may be performed, for example, as described in a document titled "Trellis-
25   coded modulation with multidimensional constellation", L.F. Wei, IEEE Trans. Inform. Theory, Vol. 33, No. 4, July 1987, the disclosure of which is incorporated herein by reference. The constellation of each multi-dimensional symbol $\underline{x}_{n-k}$ is partitioned into J_k subsets, in a set partitioning denoted $\Omega(k)$. The number J_k of subsets into which the symbols are grouped may be anywhere between one and D^M. Optionally, the subsets of the MD symbol $\underline{x}_{n-k}$ in the

partitioning $\Omega(k)$ are ordered and a specific one of the subsets is referred to by $a_{n-k}(k)$ which take on an integer between 0 and J_k-1.

Optionally, for at least one of the M-dimensional symbol subsets, the number J_k of subsets is smaller than D^M. In some embodiments of the invention, the numbers J_k are non-increasing (i.e. $J_1 >= J_2 >= ... >= J_L$). Additionally or alternatively, the multi-dimensional partitioning $\Omega(k)$ is a further partition of the subsets of $\Omega(k+1)$ for each k between 1 and L-1.

In some embodiments of the invention, subset states t_n are created (58) from all the permutations of the M-dimensional subsets:

$$t_n = [a_{n-1}(1), a_{n-2}(2), \ldots, a_{n-L}(L)]$$

The number of states of the trellis is therefore $\prod_k J_k$ k=1,2,...,L. In some embodiments of the invention, when all $J_k = D^M$ for k=1,...,L, the MD-subset trellis is equivalent to the MD-ML trellis.

Alternatively to partitioning (56) the MD constellation of each MD symbol $\underline{x}_{n-k}$ into an M-dimensional subset and creating the subset states t_n from the permutations of the M-dimensional subsets, any other method for partitioning the MD-ML trellis states p_n into subset states t_n, is used. Preferably, the partitioning is performed such that the subset-states t_n together with any subset a_n of the current transmitted MD-symbol $\underline{x}_n$ uniquely define a next subset state t_{n+1}.

In some embodiments of the invention, the subset partitioning is performed in a manner which optimizes one or more performance parameters of the equalizer performing the symbol detection. Optionally, the one or more optimized performance parameters comprise one or more of a probability of error rate, a detection error rate, a computation complexity and a space complexity. In some embodiments of the invention, the subset grouping is performed in a manner which minimizes a detection error rate of the equalizer given a maximal computation complexity and/or a maximal space complexity. Alternatively, any other tradeoff between the computation and/or space complexity and the detection error rate of the equalizer, is used.

In some embodiments of the invention, a plurality of potential subset partitionings of the MD symbols are prepared. For each of the partitionings one or more performance parameters are calculated and the grouping with the best performance parameters according to

a predetermined optimization criteria, is chosen. Various single- and multi-dimensional search methods, as known in the art, may be used to optimize the parameters and/or to select which potential subset partitionings to generate.

In some embodiments of the invention, the subset grouping includes determining one or more of the number of symbols M included in an MD symbol, the number of subset-states t_n in the MD-subset trellis, the number of subsets J_k for each MD symbol and/or the actual MD-set partitioning $\Omega(k)$ for each MD-symbol.

In some embodiments of the invention, the number of symbols M included in an MD symbol is chosen arbitrarily, e.g., M=2. If the optimization does not achieve minimal required values for the optimized parameters, a greater or smaller value of M may be chosen.

In some embodiments of the invention, the number of subset-states (t_n) of the MD-subset trellis is chosen based on computational and/or space complexity considerations and the MD-ML trellis states (p_n) included in each of the groups are chosen based on an error rate parameter. Optionally, also the number of subsets J_1 is chosen based on complexity considerations as J_1 affects the number of distinct branches 48 in the trellis. Alternatively or additionally, the number of subset-states (t_n) of the MD-subset trellis is chosen so as to achieve a desired error rate. In some embodiments of the invention, the number of subset-states (t_n) of the MD-subset trellis is chosen such that the error rate is only slightly higher than the error rate of a trellis in which each trellis state has a separate state (i.e. the MD-ML trellis). Optionally, for a given number of subset-states (t_n) of the trellis the error rate is determined for a best subset-state grouping. If the error rate is not substantially higher than the error rate for a trellis in which each trellis state has a separate state, subset-state groupings are prepared for a smaller number of subset-states (t_n) of the trellis. This procedure is repeated until a subset grouping with a desired error rate is achieved. Optionally, the number of subset-states (t_n) of the trellis is reduced by a factor of two in each repetition. Alternatively, any other reduction rate and/or method is used in progressing between repetitions.

In some embodiments of the invention, the error rate or the probability of error is determined or approximated as described hereinbelow or as described in US provisional patent application 60/159,410, filed October 14, 1999, or as described in Lior Ophir, "Multi-Dimensional Reduced State Sequence Estimation", thesis submitted towards the degree of "Master of Science in Electrical Engineering" in Tel Aviv university, October 1999, the disclosure of which documents is incorporated herein by reference.

Fig. 9 is a schematic illustration of four possible set partitionings for various channels, in accordance with an exemplary embodiment of the present invention. In some embodiments of the invention the partitioning of the of multi-dimensional sets is performed considering the matrix F_0 in hand. Due to the fact that the multi-dimensional constellation undergoes a linear

5   transformation, which is governed by the value of F0, it is possible to design the set partitioning according to this matrix. In some embodiments of the invention the goal of maximizing $d'_{min}(E'1)$ is desired. For example, for

$$F_0 = \begin{bmatrix} 1 & 0.9 \\ 0 & 1 \end{bmatrix}$$

Two possible options of two-way set partitioning are given in Fig. 9(a) Fig. 9(b). Although the

10   partitioning of Fig. 9(a) is based on the conventional two-way partitioning of the basic 2D 16-QAM constellation (that results from 2 consecutive 1D symbols each taken from a 4-PAM constellation), it is clear that the one of Fig. 9(b) is better, and if used in the 2D-RSSE for the appropriate matrix F0, it will result in better performance, due to the larger "transformed intrasubset distance", $d'_{min}(E'1)$ in the partitioning.

15   While in the partitioning of figure 9(a) we get $d'_{min}{}^{\wedge}2(E'1) = 1.01$, which is reached by the neighboring points of both subset "a" and subset "b", in figure 9(b) we get $d'_{min}{}^{\wedge}2(E'1) = 1.81$, which is also reached by the neighboring points of both subset "a" and subset "b". Specifically, for the 2D approach, when the basic input symbols are drawn from a 1D PAM constellation, it is easy to show that the worst case, in terms of $d'_{min}(E'1)$, arises when F0

20   follows

$$F_0 = \begin{bmatrix} 1 & \frac{1}{2} + i \\ 0 & 1 \end{bmatrix} \quad \forall \quad i = 0, \pm 1, \pm 2, \dots$$

in which case, both 2-way set partitioning options achieve $d'min{}^{\wedge}2(E'1) = 1.25$ (assuming a large enough 1D constellation).

Taking into account constellation boundary effects, the worst case will be when F0

25   follows i = -1,0, in which case the number of nearest neighbors in each subset will be maximized.

This case (for F0 following i = 0, and for a 4-PAM basic constellation) can be seen in figure 9(c) and 9(d). In figure 9(c) the set partitioning is based on the conventional 16-QAM set partitioning, in 9(d) a second possible set partitioning is given achieving the same value of

$d'_{min}{}^2(E'1)$ for the specific F_0 discussed in this example.

Reference is also made to Fig. 4, which is a schematic illustration of producing an MD-subset trellis 38 for an exemplary channel, in accordance with an embodiment of the present invention. Producing an MD-subset trellis 38 includes defining the subset states 44 of the MD-subset trellis and branches 48 of the MD-subset trellis, as is now described. The multi-dimensional transmitted (input) symbols $\underline{x}_n$, which in this example are two-dimensional (2D) symbols, takes values from the 4 element set:

$$\underline{x}_n \in \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix} \right\} \quad \forall n,$$

Optionally, the possible values of the multi-dimensional transmitted symbols $\underline{x}_n$ are partitioned using a 2-way 2-dimensional set partitioning as follows:

$$\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} , \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right\} , \left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} , \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

2D-ML trellis states $p_n = \{\underline{x}_{n-1}, \underline{x}_{n-2}\}$ are defined, and 2D-subset trellis states $t_n = \{a_{n-1}(1), a_{n-2}(2)\}$ are defined, representing the subsets of the two most recent symbols in the respective partitioning $\Omega(1)$ and $\Omega(2)$. If, for example, $J_1 = J_2 = 2$, and $\Omega(1)$ and $\Omega(2)$ follow the 2-way set partitioning given above, the 4 subset states of the 2D-subset trellis are generated (58) from all possible permutations of 2D-symbol subsets. For the example of Fig. 3, four 2D-subset trellis subset-states 44 are created.

Each state 44 represents the possibility that, at time n, the L last multi-dimensional symbols $\{\underline{x}_{n-1}, ..., \underline{x}_{n-L}\}$ had values included in the subsets of the specific state. For illustration purposes, the states 44 are duplicated twice, once for the current time n (states 44) and once for a following time n+1 (next states 46). From each state 44, J_1 distinct branches 48 representing the J_1 symbol groups to which transmitted MD-symbol $\underline{x}_n$ may belong, lead from current subset-states 44 to next subset-states 46. Branches 48 are created (60) by determining for each subset-state 48 and each currently transmitted multi-dimensional symbol $\underline{x}_n$, the next state 46 to which the sequence $\underline{x}_n, \underline{x}_{n-1}, ..., \underline{x}_{n-L+1}$ belongs.

In some embodiments of the invention, the created trellis is searched (62) for a given received sequence of symbols, so as to provide a sequence of detected input (transmitted) symbols. One exemplary method for searching the trellis is described hereinbelow with reference to Fig. 6.

In some embodiments of the invention, the method of Fig. 2 for generating an MD-subset trellis is performed by a manufacturer of an equalizer in manufacturing and/or calibrating the equalizer. Alternatively or additionally, one or more of the acts of the method of Fig. 2 are performed by the equalizer periodically and/or each time a decoding is to be

5   performed. This alternative may be used, for example, when the characteristics of channel 24 change substantially within short periods, e.g., when receiver 26 comprises a cellular receiver. The method of Fig. 2 may be performed, for example, each time the receiver is turned on and/or when the receiver moves between base stations.

In some embodiments of the invention, using the method of Fig. 2 or any other method,

10   an MD-subset trellis is formed in which the movement between a current subset state and a next subset state is based on a plurality of consecutively transmitted MD-symbols and at least some of the states of the MD-subset trellis represent a plurality of MD symbol sequences. The use of such a trellis provides in many cases a substantially lower decoding error rate relative to other trellises, for a given computational complexity level.

15   Fig. 5 is a schematic block diagram of a data structure 70 managed by an equalizer, in accordance with an embodiment of the present invention. Data structure 70 includes a description 72 of a MD-subset trellis which is used by the equalizer. Trellis description 72 includes a listing 74 of the MD-subsets included in each of the subset states of the MD-subset trellis for each subset state or the MD-ML states (p_n) included in the MD-subset states (t_n). In

20   addition, trellis description 72 includes a branch listing 76 which lists for each subset-state of the MD-subset trellis, the subset-states to which branches from the subset-state lead in a following time (n+1). Branch listing 76 also lists for each branch the corresponding MD-symbol or subset. Alternatively, the corresponding MD symbol or subset is listed separately. Alternatively or additionally, branch listing 76 lists for each subset-state, the subset-states from

25   which the subset-state has leading branches in a previous time.

In addition, data structure 70 includes a register array 78 which includes a register 80 for each subset-state of the trellis, for a plurality of times δ. Register 80 comprises a sequence field 82 which stores a most likely symbol sequence (i.e., a path history) from the group of symbol sequences of the respective subset-state, a cost field 84 which represents the chances

30   that the symbol sequence listed in sequence field 82 was the sequence transmitted by transmitter 22, given the signals r_n received by receiver 26 and a previous subset-state field 86 which indicates the previous time subset-state from which the current subset-state was

reached. Optionally, the cost field is used only for the current subset-states. Optionally, field 86 is not used, and decisions are made according to sequence field 82. Optionally, for states which represent only a single symbol sequence, sequence field 82 is not used. Alternatively, for states which represent only a single symbol sequence, register 80 does not include sequence field 82.

In some embodiments of the invention, data structure 70 also includes a function description 88 describing the model of channel 24.

Fig. 6 is a flowchart of the acts performed by an equalizer in detecting received signals, in accordance with an embodiment of the present invention. The equalizer receives (100) a sequence of M received symbols $\underline{r}_n = \{r_{n+M-1}, ..., r_n\}$, M being the number of symbols required to perform a transition in the trellis of description 72, i.e., the dimensionality of the MD model.

If (102) all the groups of multi-dimensional symbol $\underline{x}_n$ represent only a single MD-symbol, i.e., $J_1 = D^M$, the equalizer computes (104) for each of the branches of the trellis, as listed in branch listing 76, a cost of passing on that branch, given the channel function description 88 and the received MD-sample $\underline{r}_n$. Optionally, the cost of passing the branch is defined as the Euclidean distance between the received MD-sample $\underline{r}_n$ and the calculated result of passing the transmitted symbols represented by the MD-subset state from which the branch leaves and the MD-symbol represented by the branch through channel 24 as modeled by function description 88 (Fig. 5). In some embodiments of the invention, the branch costs are calculated using the following equation:

$$B_{\tau_{l,i}} = \left\| \underline{r}_n - (\hat{p}_n(t_n), F) - F_0 \underline{x}_n \right\|^2$$

in which $B_{\tau l,i}$ is the cost of the transition $\tau_{l,i}$ leading from state i to state l, $\underline{x}_n$ is the MD-input-symbol corresponding to the branch, $\hat{P}_n(t_n)$ represent the L most recent M-dimensional symbols stored in the path history associated with the subset-state t_n (field 82 of the register 80 of state i), F_0 is the first matrix of the MD representation of the channel 24, $F = [F_1, F_2, ... , F_L]$ are the L matrices representing the channel 24, and "(,)" denotes the inner product (of the subset-state (using the path history) and the L channel matrices) defined by:

$$(\underline{p}_n, F) = \sum_{k=1}^{L} F_k \underline{x}_{n-k}$$

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The symbol ‖ ‖ denotes the Euclidean distance. Alternatively, any other distance metric is used, such as any simplified approximation of the Euclidean distance.

If (102), however, some of the subsets of multi-dimensional symbol $\underline{x}_{n-1}$ represent a plurality of MD-symbols, parallel transitions may exist in the MD-subset-trellis (i.e. transitions that start from a common state and end in a common state may represent different MD-symbols $\underline{x}_n$). If, for example, all the subsets include the same number of MD-symbols, each distinct branch represents D^M/J_1 parallel transitions. Optionally in these cases, the equalizer computes the cost of each branch by finding the minimal cost of each group of parallel transitions. In some embodiments of the invention, the equalizer computes (106) the cost of the branch for each MD-symbol in the parallel transitions (MD-subset) represented by the branch, and chooses (108) the lowest cost for passing the branch. In addition to storing the lowest cost (in field 84 (Fig. 5), the MD-symbol which resulted in the lowest score is stored in field 82 (Fig. 5). In some embodiments of the invention, the branch metric is calculated according to :

$$B_{\tau_{l,i}} = \min_{\underline{x}_n \in \mathcal{R}_j(1)} \left\| \underline{r}_n - (\hat{p}_n(t_n); F) - F_0 \underline{x}_n \right\|^2$$

where the minimum is computed over all MD-symbols $\underline{x}_n$ in each subset $R_j(1)$ of $\Omega(1)$ for ($1 <= j <= J_1$), corresponding to the transition $\tau_{l,i}$.

Alternatively, the MD-symbol which results in the lowest score is selected using slicing methods, without explicitly calculating each transition cost. The transition cost is then calculated only for the transition with the lowest cost for each group of parallel transitions.

Using the calculated branch costs, the equalizer calculates (110) new state costs for each of the states 44 of the trellis. For every state, the path with the smallest metric that reaches that state is found. The new state metric is a sum of the previous state's metric and the metric of the branch leading from the previous to the current state. For each state, the equalizer stores in registers 80 of time n, the new state cost (in field 84), the MD-symbol of the selected transition (in field 82) and the previous state from which the branch entering the state came (in field 86). In mathematical notation, the state's new metric is calculated according to:

$$\mathcal{P}_l(n) = \min_{i \in \mathcal{M}_l}\{\mathcal{P}_i(n-1) + B_{\tau_{l,i}}\} \quad \forall l \in \mathcal{M} \tag{49}$$

where $P_l(n)$ is the metric of state l at the n'th instant. M_l is the set of all states that have

branches leading to state 1, and *M* is the set of all states.

In some embodiments of the invention, the equalizer then finds (112) the state with the lowest cost and traverses the trellis backwards (using the best path history of the selected state stored in fields 82) for a predetermined number of steps (δ), and the MD-symbol stored in the backward traversed field 82, e.g., $\underline{x}_{n-\delta}$, is provided (114) at the output of the equalizer. In some embodiments of the invention, the method of Fig. 6 is repeated a plurality of times for an entire received symbol sequence. In an exemplary embodiment of the invention (δ) is between 2L and 4L.

Alternatively to finding (112) the lowest cost state and providing an output MD-symbol for each trellis iteration performed, a plurality of symbols are provided every predetermined number of trellis iterations. In this alternative, the equalizer finds the state with the lowest cost every predetermined number of iterations, traverses back from the state with the lowest cost δ steps and then provides as output a sequence of MD-symbols. Optionally the equalizer may use some other decision rule to determine the state from which to begin to traverse backwards.

In some embodiments of the invention, the provided MD-symbol (or symbols) are converted to one dimensional symbols before they are provided by the equalizer.

Alternatively to computing (106), for each transition, the cost of passing the branch for each MD-symbol in each subset represented by the branch and choosing (108) to represent the branch the MD-symbol resulting with the lowest cost, the equalizer determines the MD-symbol which will have the lowest cost using a slicing method known in the art and calculates the cost of the branch only for the determined sequence.

As described above, the specific MD-RSSE trellis used may be chosen responsive to the computation and/or space complexity of using the specific trellis. In some embodiments of the invention, the specific MD-RSSE used is chosen based on the performance characteristics of the MD-RSSE. In some embodiments of the invention, the performance is estimated using simulations. Alternatively or additionally, the complexity of the specific MD-RSSE trellis is estimated as is know described.

In some embodiments of the invention, the space complexity required for performing the detection is estimated as being proportional to the number of states in the trellis. For the MD-Subset trellis, the number of states is:

$$N = \prod_{k=1}^{L} J_k \tag{60}$$

and the space complexity of the MD-RSSE is o(N).

The computational complexity of the MD-RSSE can optionally be estimated as being of the order of the number of states in the trellis, multiplied by the number of branches leaving each state.

In an alternative embodiment of the invention, the computational complexity is represented by one or more of the following three different categories, which include the number of branch metric calculations, the number of summations needed for path metric computation, and the number of comparisons. Without loss of generality, in this discussion, for simplicity reasons, we shall consider symmetric set partitioning, such that all subsets $R_j(1)$ ($1 <= j <= J1$) in the partitioning $\Omega(1)$ consist of the same number of elements. The number of states in the MD-S trellis follows N, whereas the number of branches to and from each state is D^M.

When $J_1 < D^M$ these branches may be divided into J_1 groups of parallel transitions, each consisting of D^M/J_1 branches (in the symmetric case).

As described above, in some embodiment of the invention, each iteration of the method of Fig. 6 used to search the MD-S trellis includes:

(a) If $J1 = D^M$ use step (a1), if $J\_1 < D^M$ use step (a2). (a1) D^M branch metrics are calculated for each state, leading to $N*D^M$ calculations. (a2) Using brute force methods, D^M branch metrics are calculated for each state, and delay-free decisions are made between every $D^M/J1$ symbols, for each subset in the partitioning $\Omega(1)$, leading to $N*D^M$ calculations, and $N*J1(D^M/J1-1)$ comparisons. (b) $N*J1$ path metrics are calculated, as the sum of previous path metrics and branch metrics. (c) A minimal path metric is found out of $J1$ metrics, for each of the N states, leading to $N(J1-1)$ comparisons. (d) One minimal state is chosen out of N states, leading to N-1 comparisons.

In some embodiments of the invention, when $J1 = D^M$, and when $J1 < D^M$ the total number of operations is as follows:

Number of branch metric calculations: $N*D^M$

Number of summations: $N*J1$

Number of comparisons: $N*D^M - 1$

Notice the linear dependency of the total number of operations on the number of states in the trellis N.

In some embodiments of the invention it is desirable to choose J_1 as small as possible,

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without causing d'min to become too small (through early merging error events of length 1) which may lead to a degradation in performance. Also notice the exponential dependency on the size of M, the dimensionality of the multi-dimensional approach used.

On the other hand, each iteration of the MD-RSSE results in estimating an M-dimensional symbol (i.e. optionally M consecutive symbols), and thus the number of operations at each iteration should be divided by M, when compared with classical, one-dimensional methods.

In some embodiments of the invention, the branch metric calculation is performed by finding the minimum squared Euclidean distance between pairs of points in the M-dimensional space. Optionally other types of distances may be used. A first point in the pair is defined as:

$$z'_n \stackrel{\text{def}}{=} r_n - (\hat{p}_n(t_n), F) \tag{61}$$

Point z'_n is associated with the received multi-dimensional signal r_n , and the path history of the subset state t_n . A second point in the pair is defined as:

$$x'_n \stackrel{\text{def}}{=} F_0 x_n \tag{62}$$

which is a linear transformation of all MD-constellation points in the respective multi-dimensional subset of $\Omega(1)$. Optionally, the form of F_0 is used to derive an efficient way of finding the closest multi-dimensional subset point, even if it is difficult to use slicing in order to find the closest multi-dimensional symbol in the linearly transformed subset. Particularly, this method may be used when the partitioning $\Omega(1)$ has a symmetry. Optionally, when F_0 is of the form:

$$F_0 = \begin{bmatrix} f_0 & f_1 & \cdots & f_{M-2} & f_{M-1} \\ 0 & f_0 & \cdots & f_{M-3} & f_{M-2} \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & f_0 \end{bmatrix} \tag{63}$$

the closest multi-dimensional transformed subset point is found in an iterative way. The M'th dimension is scaled by a factor f_0 , and a decision is made in the M'th dimension using a slicing operation.

Denoting the i'th element of a vector v as $v^{(i)}$, once the M'th element of x_n, namely $x_n^{(M)}$ is known, it is used to find the (M-1)'th element, which is transformed to be:

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$$x_n^{\prime(M-1)} = f_0 x_n^{(M-1)} + f_1 x_n^{(M)} \tag{G4}$$

using a simple slicing operation. In some embodiments of the invention, this procedure is repeated for all M dimensions, yielding for each state a total of only J_1 explicit branch metric computations, one for each multi-dimensional subset of the partitioning $\Omega(1)$. Optionally, the

5   temporary results, i.e. the branch metrics of each dimensions, used for the iterative slicing procedure, can be used for the computation of the branch metric, without re-computing them.

Alternatively, any other method, such as described in L.F. Wei, "Trellis-coded modulation with multi-dimensional constellation", IEEE Trans. Inform. Theory, Vol. 33, No. 4, July 1987, is used to determine the symbol sequence with the lowest cost without

10   calculating the cost for each of the symbol sequences of the branch. Optionally, the partitioning of a multi-dimensional lattice is performed in an iterative manner. That is, the partitioning of a 2M-dimensional lattice is based on the partitioning of the constituent M-dimensional lattices, which is in turn based on the partitioning of the constituent M/2-dimensional lattices. Based on dividing the basic (optionally 1D or 2D) constellation into

15   subsets, multi-dimensional types are constructed by concatenating basic subsets, while multi-dimensional subsets, sub-families and families are constructed by grouping together several multi-dimensional types, subsets and sub-families respectively. Following this method of constructing the multi-dimensional partitioning, a simplified maximum-likelihood decoding algorithm is given, in which the point in each multi-dimensional subset closest to the received

20   multi-dimensional point is also found in an iterative way. The multi-dimensional received signal is divided into several constituent signals lying in a space with the basic dimensionality (say M). Each received 2M-dimensional point is divided into a pair of M-dimensional points. The closest point in each 2M-dimensional subset and its associated metric is found based on the point in each of the M-dimensional subsets which is closest to the corresponding received

25   M-dimensional point and its associated metric. The M-dimensional subsets, are optionally those that were used to construct the 2M-dimensional subsets. This process is optionally performed iteratively to obtain the closest point in each 2M-dimensional subset and its associated metric, based on the closest point in each of the basic (optionally 1D or 2D) subsets and its associated metric.

30   Alternatively or additionally, when the multi-dimensional partitioning $\Omega(1)$ used for generating the MD-subset trellis is similar to the MD-set partitioning described in L.F. Wei,

"Trellis-coded modulation with multi-dimensional constellation", IEEE Trans. Inform. Theory, Vol. 33, No. 4, July 1987, iterative methods are used as described above, in order to reduce the number of calculations needed for each iteration of the method of Fig. 6. Alternatively, a combination of some or all of the methods described above is used.

5   Referring in more detail to determining a probability of error parameter for use in selecting a partitioning to subsets, following is an optional procedure for calculating a parameter of a probability of symbol error.

An error event E is said to occur, extending from time k_1 to k_{1+l}, if the estimated state sequence $\{\hat{p}_n(\hat{t}_n)\}$ is equal to the correct state sequence $\{P_n(t_n)\}$ at times k_1 and k_{1+l} but

10   nowhere in between. For that we define the error sequence e, corresponding to E as:

$$e = \left[\underline{e}_{k_1}, \underline{e}_{k_1+1}, \cdots, \underline{e}_{k_1+l-1} \right]$$

where each element of e follows:

$$\underline{e}_k \overset{\text{def}}{=} \underline{x}_k - \hat{\underline{x}}_k$$

Each error sequence e corresponding to the error event E obeys the three rules: (a) The element

15     $\underline{e}_{k1}$ is nonzero. (b) The last L elements, $\underline{e}_{k1+l-L}$ through $\underline{e}_{k1+l-1}$ satisfy the "subset merging condition", and so do the L elements prior to the beginning of e. This means that $\underline{e}_{k1+l-L-1+k}$ is equal to ($\underline{x}_{k1+l-L-1+k} - \underline{x}'_{k1+l-L-1+k}$) where the correct multi-dimensional input symbol $\underline{x}_{k1+l-L-1+k}$ and the estimated symbol $\underline{x}'_{k1+l-L-1+k}$ belong to the same subset in the partitioning $\Omega(k)$ for $k = 1,2,...,L$. (c) No L elements of e prior to the last ones satisfy the

20   "subset merging condition".

When MD-ML states are used, two paths merge only if their last L elements are the same. When MD subset states are used, paths merge in the MD-Subset trellis if the subsets of their last L elements are the same. This difference can be seen in rule (b) above. Moreover, generally an error event in the MD-RSSE (i.e., the method in which the MD-subset trellis is

25   used) may occur, even if the estimated subset state sequence $\{t'_n\}$ is equal to the correct state sequence $\{t_n\}$. When $J_k = D^M$ for all k, there is no difference between the MD-RSSE, and the MD-MLSE (the method in which the MD-ML trellis is used), but generally the MD-S (MD-subset) trellis will have a set of what may be called "early merging error events" which are not present in the MD-ML trellis. On the other hand, some error events in the MD-ML trellis, may

30   not belong to the set of error events of the MD-S trellis. This may occur for instance for MD-

ML trellis error events for which L elements, prior to the last ones, satisfy rule (c) above, which holds a weaker condition than the corresponding one for MD-MLSE.

When $J_1 < D^M$, parallel transitions appear in the MD-S trellis. In this case, error events of length 1, associated with the parallel transitions are part of the "early merging error events". Another difference between MD-RSSE and MD-MLSE concerns the phenomenon of error propagation. In the MD-RSSE, when the correct path is discarded in the subset trellis, it is possible, due to the use of decision feedback, to have error propagation. It is possible to analyze the effect of error propagation analytically (e.g. using state machine models), or assess the effect of error propagation through simulations. In the analysis given here the probability of a first error event occurring at some time k_1 is analytically analyzed. This gives close approximation to the symbol error probability.

Following the definition of the error sequence e related to the input symbols, an error sequence e_y associated with the signal sequence $\{y_n\}$ is defined as:

$$e_y = \left[\underline{e_{y_{k_1}}}, \underline{e_{y_{k_1+1}}}, \cdots, \underline{e_{y_{k_1+l-1}}} \right]$$

where each element of e_y follows:

$$\underline{e_{y_k}} \stackrel{\text{def}}{=} \underline{y_k} - \underline{\hat{y}_k}$$

The squared distance $d^2(e)$ of an error event e is defined as:

$$d^2(e) \stackrel{\text{def}}{=} \|e_y\|^2 = \sum_{k=k_1}^{k_1+l} \left\| \underline{e_{y_k}} \right\|^2$$
$$= \sum_{k=k_1}^{k_1+l} \left\| \sum_{j=0}^{L} F_j \underline{e_{k-j}} \right\|^2$$
$$= \sum_{k=k_1}^{k_1+l} \left\| F_0 \underline{e_k} + \sum_{j=1}^{L} F_j \underline{e_{k-j}} \right\|^2$$

where $\underline{e_k} = \underline{0}$ for $k_1-L <= k <= k_1-1$, following condition (b) above, and the fact that we deal with a first error event. The number of errors in the error sequence e associated with the error event E is also defined. $w(e)$ is defined, for example, as the Hamming weight of the error event E, with which the error sequence e is associated.

For the first error event E to occur, three sub-events typically occur:

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- $\mathcal{E}_1$: At time k_1 $\{\hat{p}_{k_1}(\hat{t}_{k_1})\} = \{p_{k_1}(t_{k_1})\}$

- $\mathcal{E}_2$: Between k_1 and $k_1 + l - 1$ the input symbols $x_{k_1}, x_{k_1+1}, \ldots, x_{k_1+l-1}$ must be such that when added to the error sequence e, they result in an allowable sequence.

- $\mathcal{E}_3$: For $k_1 \leq k \leq k_1 + l$, the sequence $\{\hat{x}_k\}$ must have greater likelihood than $\{x_k\}$.

For E3 to occur we get:

$$\Pr\{\mathcal{E}_3\} = Q\left(\frac{d(e)}{2\sigma}\right)$$

As mentioned herein, E3 can occur without actually leaving the correct subset state sequence
$\{t_n\}$ in the MD-S trellis.

The probability of the sub-event E2 depends only on the statistical properties of the input sequence, and on the value of the error sequence e. Furthermore it is independent of E1 and E3. Let us consider a PAM (pulse amplitude modulation) case known in the art where the input sequence is independent and equiprobable, taken from an alphabet of size D (assumed even) $\{-(D-1), \ldots, -1, +1, +3, \ldots, D-1\}$, then:

$$\Pr\{\mathcal{E}_2\} = \prod_{k=k_2}^{k_2+l-1} \prod_{j=1}^{M} \frac{2D - |e_{k,j}|}{2D} \tag{50}$$

Optionally, input sequences other than PAM can be dealt with in a similar manner. The probability that E1 is not true, is of the order of the error probability, and therefore:

$$\Pr\{\mathcal{E}_1|\mathcal{E}_3\}$$

is well approximated as well as upper bounded by 1, for reasonably low symbol error probabilities. Altogether the probability of a particular error event E, to which the error sequence e is associated, is given by:

$$\begin{aligned}
\Pr\{\mathcal{E}\} &= \Pr\{\mathcal{E}_3\}\Pr\{\mathcal{E}_2\}\Pr\{\mathcal{E}_1|\mathcal{E}_3\} \\
&\leq \left[Q\left(\frac{d(e)}{2\sigma}\right)\right]\left[\prod_{k=k_2}^{k_2+l-1}\prod_{j=1}^{M}\frac{2D-|e_{k,j}|}{2D}\right]
\end{aligned} \tag{51}$$

Let E' be the set of all first error events E, starting at time k_1 . Then using the union bound, the probability that a first error event starts at time k_1 is upper bounded by

$$\Pr\{E'\} \leq \sum_{\mathcal{E} \in E'} \Pr\{\mathcal{E}\} \tag{52}$$

let us separate the set E' into several distinct sets: (a) The set E'_{ML} of all error events that belong to the set E of all error events E, starting at time k_1 in the MD-ML trellis. (b) The set E'_{12} of all the "early merging error events" E, starting at time k_1, which are not present in the MD-ML trellis. When $J_1 < D^M$, the set E'_{12} may be represented by the disjoint union of E'_{12} = E'_1 U E'_2 where: (b1) The set E'_1 represents all early merging error events of length 1, associated with parallel transitions in the MD-S trellis, at time k_1. (b2) The set E'_2 represents all other early merging error events in the MD-S trellis, starting at time k_1.

Letting G be the set of all d(e) , and for each d(e) belonging to G letting E'_d be the subset of all error events for which d(e) = d, the probability of a symbol error can be upper bounded by:

$$\Pr\{e\} \leq \sum_{\varepsilon \in E'} w(e)\Pr\{\mathcal{E}\}$$
$$\leq \sum_{d \in G} Q\left(\frac{d(e)}{2\sigma}\right) \sum_{\varepsilon \in E'_d} w(e) \left[\prod_{k=k_2}^{k_2+l-1} \prod_{j=1}^{M} \frac{2D - |c_{k,j}|}{2D}\right] \tag{53}$$

Due to the exponential nature of the Q-function, the expression is dominated at moderate SNR by the term involving the minimum value d'_{min} of d(e) , for all error sequences e that correspond to the error events E belonging to E'.

d'_{min} can be written as:

$$d'_{min} = \min(\min_{\varepsilon \in E'_{ML}} d(e), \min_{\varepsilon \in E'_1} d(e), \min_{\varepsilon \in E'_2} d(e))$$
$$= \min(d_{min}, \min_{\varepsilon \in E'_1} d(e), \min_{\varepsilon \in E'_2} d(e)) \tag{54}$$

Let us now consider the early merging error events of length 1. The minimum distance between all parallel transitions should be calculated according to all the possible multi-dimensional intra-subset errors $\underline{e}_1$ for the partitioning $\Omega(1)$ so that

$$d_{min}(E'_1) \overset{def}{=} \min_{\varepsilon \in E'_1} d(e)$$
$$= \min_{\underline{e}_1} \|F_0 e_1\|^2 \tag{55}$$

Altogether:

$$d'_{min} = \min(d_{min}, d_{min}(E'_1), d_{min}(E'_2)) \tag{56}$$

where d'_{min} (E'$_1$) is given above and d'_{min} (E'$_2$) is the minimum distance among the remaining early merging error events that are longer than one multi-dimensional symbol interval. Therefore care should be taken as to the partitioning $\Omega(1)$ and to the value of J_1 ,

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taking into consideration also the value of F_0 . Now, based on the probability of first error events (i.e. neglecting the effect of error propagation) the symbol error probability can be approximated by

$$Pr\{e\} \approx K_{d'_{min}} Q\left(\frac{d'_{min}}{2\sigma}\right) \tag{57}$$

5 where for the case of PAM, as discussed above

$$K_{d'_{min}} \stackrel{def}{=} \sum_{\varepsilon \in E_{d'_{min}}} w(e) \left[\prod_{k=k_1}^{k_1+l-1} \prod_{j=1}^{M} \frac{2D - |e_{k,j}|}{2D} \right] \tag{58}$$

Therefore, for approximating the symbol error probability for a given channel and a given transmission scheme and noise variance, we may determine the value of d'$_{min}$, and then use

10 $K_{d'min}$. Optionally the noise variance (sigma) may also be approximated, if not known, by some method known to the art. Alternatively, for the case of QAM (Quadrature amplitude modulation) transmission, the same method may be performed, only $K_{d'min}$ is found to be:

$$K_{d_{min}} \stackrel{def}{=} \sum_{\varepsilon \in E_{d_{min}}} w(e) \left[\prod_{k=k_1}^{k_1+l-1} \prod_{j=1}^{M} \left(\frac{2D - |Re\{e_{k,j}\}|}{2D}\right) \left(\frac{2D - |Im\{e_{k,j}\}|}{2D}\right) \right]$$

In some embodiments of the invention, Pr{e} is used as a criteria parameter in

15 selecting a subset partitioning in the method of Fig. 2.

Following is an example of using the MD-RSSE, for a binary transmission through a finite impulse response (FIR) filter having the following characteristic:

$$f_0 = 1$$

$$f = [f_1, f_2, f_3, f_4] = [0.1, -0.1, 0.2, 0.8]$$

Leading to the following input-output relation:

$$\begin{aligned} r_n &= \sum_{k=0}^{4} f_k x_{n-k} + w_n \\ &= x_n + 0.1 x_{n-1} - 0.1 x_{n-2} + 0.2 x_{n-3} + 0.8 x_{n-4} + w_n \end{aligned}$$

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The energy of the filter is:

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$$\|f\|^2 = \sum_{k=0}^{5} f_k^2 = 1.7$$

such that in the output sequence, only 59% of the energy of the input symbols will be embedded in the cursor term, while 41% will be embedded in ISI terms. The severity of the channel can also be seen from its frequency response having attenuation of over 20dB. Examination of the d_{min} property of this channel, with a binary input, yields the following:

$$d_{min}^2 = 6.8$$

corresponding to the error sequence of length 5 having the form:

$$e_{min} = [\pm 2, 0, 0, 0, 0]$$

In a 2-dimensional approach:

$$\underline{x}_0 = \begin{bmatrix} x_1 \\ x_0 \end{bmatrix}, \underline{x}_1 = \begin{bmatrix} x_3 \\ x_2 \end{bmatrix}, \underline{x}_2 = \begin{bmatrix} x_5 \\ x_4 \end{bmatrix}, \cdots$$

$$\underline{x}_n \in \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right\} \quad \forall \, n,$$

$$\underline{r}_0 = \begin{bmatrix} r_1 \\ r_0 \end{bmatrix}, \underline{r}_1 = \begin{bmatrix} r_3 \\ r_2 \end{bmatrix}, \underline{r}_2 = \begin{bmatrix} r_5 \\ r_4 \end{bmatrix}, \cdots$$

The three 2*2 matrices (following L = 2) describing the channel, are:

$$F_0 = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix}, F_1 = \begin{bmatrix} -0.1 & 0.2 \\ 0.1 & -0.1 \end{bmatrix}, F_2 = \begin{bmatrix} 0.8 & 0 \\ 0.2 & 0.8 \end{bmatrix}$$

and the relation between the input and output is modeled by:

$$r_n = \sum_{k=0}^{2} F_k x_{n-k} + w_n$$

$$= \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} x_n + \begin{bmatrix} -0.1 & 0.2 \\ 0.1 & -0.1 \end{bmatrix} x_{n-1} + \begin{bmatrix} 0.8 & 0 \\ 0.2 & 0.8 \end{bmatrix} x_{n-2} + w_n$$

In the MLSE model, trellis states are optionally defined as:

$$p_n = [x_{n-1}, x_{n-2}]$$

Since each 2-dimensional element of the state vector can take one of 4 values, the 2D-ML trellis has 16 states, with 4 transitions to and from each state. In some embodiments of the invention, the Viterbi algorithm (VA) is used to search the 2D-ML trellis as described above.

In some embodiments of the invention, a 2-way 2-dimensional set partitioning in which $J_1, J_2 = 2$ is used and $\Omega(1)$ and $\Omega(2)$ are chosen as:

$$\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right\}, \left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

The subset trellis of a 2-dimensional sequence at time n is defined as: $t_n = [a_{n-1}(1), a_{n-2}(2)]$ representing the subsets of the two most recent symbols in the respective partitioning $\Omega(1)$ and $\Omega(2)$. Since each element of t_n can take on one of two values, the total number of states in the subset trellis is 4, with $J_1 = 2$ transitions to and from each state. It is noted that each branch represents 2 parallel transitions that end in the same state.

The possible 2-dimensional intra-subset errors for the partitioning $\Omega(1)$ are:

$$\varepsilon_{11} = \pm \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad \varepsilon_{12} = \pm \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

which lead to error events of length 1, associated with parallel transitions having the following values of d'^2:

$$d'^2(\varepsilon_{11}) = 8.84 > d^2_{min}$$

$$d'^2(\varepsilon_{12}) = 7.24 > d^2_{min}$$

Thus:

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$$d'^2_{min}(E'_1) = 7.24 > d^2_{min}$$

and the 2D-RSSE performance shall not be dominated the parallel transitions in the search of the subset trellis. A close examination of the subset trellis reveals that

$$d'^2_{min} = d^2_{min} = 6.8$$

5   which is achieved by the 2-dimensional error events:

$$e_{min1,2} = \left\langle \pm \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\rangle$$

$$e_{min3,4} = \left\langle \pm \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\rangle$$

meaning that in this particular example no severe degradation is expected in comparison to the full 16-state MLSE due to early merging error events. Some degradation is expected, however, in this particular example due to the larger number of error events having the d_{min} , i.e.,

10    $K_{d'min} = 2$ compared to $K_{dmin} = 1$ for the case of the full 16-state MLSE.

The above described scheme performs very closely to the full MLSE (having 16 states), and is superior over the prior art RSSE/DFSE methods by 2 dB at moderate to high SNR, even though it has less complexity. The above analysis shows that for the case discussed here, the method of the present invention outperforms tremendously the prior art methods, in terms of

15   performance/complexity tradeoff.

Following is another example of using the MD-RSSE, for a transmission using a 16-QAM constellation through a finite impulse response (FIR) filter having the following characteristic:

$$f_0 = 1$$

$$f = [f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8] = [0.05, 0.10, 0.20, 0.05, 0.30, 0.10, 0.20, 0.75]$$

20   Leading to the following input-output relation:

$$\begin{aligned} r_n &= \sum_{k=0}^{8} f_k x_{n-k} + w_n \\ &= x_n + 0.05 x_{n-1} - 0.1 x_{n-2} + 0.2 x_{n-3} + 0.05 x_{n-4} + 0.3 x_{n-5} + 0.1 x_{n-6} \\ &\quad + 0.2 x_{n-7} + 0.75 x_{n-8} + w_n \end{aligned}$$

The energy of the filter, is:

$$\|f\|^2 = \sum_{k=0}^{8} f_k^2 = 1.7373$$

such that in the output sequence, only 57% of the energy of the input symbols will be embedded in the cursor term, while 43% will be embedded in ISI terms. The severity of the channel can also be seen from its frequency response having attenuation of almost 40 dB. For the case discussed here, the full MLSE has high complexity due to the large signal constellation and the long impulse response of the channel. It is interesting to analyze it, in order to achieve theoretical lower bounds on the expected performance of any scheme.

An examination of the d_{min} property of this channel, with a 16-QAM constellation, yields the following:

$$d_{min}^2 = 1.758$$

corresponding to the four error sequences of length 9 having the form:

$$e_{min1} = [\pm 1, 0, 0, 0, 0, 0, 0, 0, 0]$$

$$e_{min2} = [\pm j, 0, 0, 0, 0, 0, 0, 0, 0]$$

Let us now adopt a 2-dimensional approach, leading to the following definitions:

$$\underline{x}_0 = \begin{bmatrix} x_1 \\ x_0 \end{bmatrix}, \underline{x}_1 = \begin{bmatrix} x_3 \\ x_2 \end{bmatrix}, \underline{x}_2 = \begin{bmatrix} x_5 \\ x_4 \end{bmatrix}, \cdots$$

is the 2-dimensional input sequence. Since the basic input symbols where taken from a 2-dimensional 16-QAM constellation, this scheme may be interpreted as 4-dimensional.

The input 2-element vectors are taken from the 4-dimensional constellation having 256 possible values. Each constituent 2-dimensional element of $\underline{x}_n$ is taken from the 16-QAM constellation of figure 8. The 2-dimensional output sequence is:

$$\underline{r}_0 = \begin{bmatrix} r_1 \\ r_0 \end{bmatrix}, \underline{r}_1 = \begin{bmatrix} r_3 \\ r_2 \end{bmatrix}, \underline{r}_2 = \begin{bmatrix} r_5 \\ r_4 \end{bmatrix}, \cdots$$

where again, each element is taken from a 2-dimensional plain, and

$$F_0 = \begin{bmatrix} 1 & 0.05 \\ 0 & 1 \end{bmatrix}, F_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.05 & 0.1 \end{bmatrix}, F_2 = \begin{bmatrix} 0.05 & 0.3 \\ 0.2 & 0.05 \end{bmatrix};$$

$$F_3 = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.1 \end{bmatrix}, F_4 = \begin{bmatrix} 0.75 & 0 \\ 0.2 & 0.75 \end{bmatrix}$$

are the five 2*2 matrices (following L = 4) describing the channel.

Using the above definitions, a simple relation between input and output may be defined as:

$$r_n = \sum_{k=0}^{4} F_k x_{n-k} + w_n$$

$$= \begin{bmatrix} 1 & 0.05 \\ 0 & 1 \end{bmatrix} x_n + \begin{bmatrix} 0.1 & 0.2 \\ 0.05 & 0.1 \end{bmatrix} x_{n-1} + \begin{bmatrix} 0.05 & 0.3 \\ 0.2 & 0.05 \end{bmatrix} x_{n-2} + \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.1 \end{bmatrix} x_{n-3}$$

$$+ \begin{bmatrix} 0.75 & 0 \\ 0.2 & 0.75 \end{bmatrix} x_{n-4} + w_n$$

where w_n represents a 2-dimensional complex noise sequence. Let us first view the construction of the 2-dimensional MLSE.

The trellis states shall be defined as:

$$p_n = [x_{n-1}, x_{n-2}, x_{n-3}, x_{n-4}]$$

since each 2-dimensional element of the state vector can take one of 256 values, the 2D-ML trellis has 256^4 states, with 256 transitions to and from each state, and is enormously complex to implement. Theoretically, a VA can be used to search the 2D-ML.

Optionally a 2-way 4-dimensional set partitioning, based on Wei, known in the art may be used. The partitioning is based on the partitioning of the 2-dimensional 16-QAM constellation into 4 subsets as shown in Fig. 8, which is identical to the 4-way Ungerboeck set partitioning of the 16-QAM constellation. Sixteen 4-dimensional types are created using the concatenation of two 2-dimensional subsets, as given in the following table:

4D family	4D subset	4D types
1	1	(a,a)
		(b,b)
	2	(c,c)
		(d,d)
	3	(a,b)
		(b,a)
	4	(c,d)
		(d,c)
2	5	(a,c)
		(b,d)
	6	(c,b)
		(d,a)
	7	(a,d)
		(b,c)
	8	(c,a)
		(d,b)

Eight 4-dimensional subsets are generated by grouping each two 4-dimensional types together, keeping a squared intrasubset distance of:

$$\Delta_s^2 = 4.$$

Next, two 4-dimensional families are generated by grouping four 4-dimensional subsets together. The squared intra-family distance is reduced to:

$$\Delta_f^2 = 2.$$

The two 4-dimensional families are used in order to generate the MD-RSSE. Notice that this 4-dimensional partitioning may be used for any basic rectangular 2-dimensional constellation.

Letting J1 = J2 = J3 = J4 = 2, so Omega(1) through Omega(4) follow the 4-dimensional two-family partitioning of the table. This allows us to define the subset trellis of a 2-dimensional sequence at time n as:

$$t_n = [a_{n-1}(1), a_{n-2}(2), a_{n-3}(3), a_{n-4}(4)]$$

representing the subsets of the four most recent symbols in the respective partitioning Omega(1) through Omega(4). Since each element of t_n can take one of two values, the total number of states in the subset trellis is 16, with J1 = 2 distinct transitions to and from each state. Notice also the existence of 128 parallel transitions that start in a common state and end in a common state.

Using the VA to search the subset trellis we get the 16-state 2-dimensional RSSE (2D-RSSE). Going over all possible 2-dimensional intrasubset errors $\underline{e}1$ for the partitioning Omega(1), reveals that:

$$d'^2_{min}(E'_1) = 1.9025 > d^2_{min}$$

where the minimum is attained for

$$\underline{e}_{11} = \pm \begin{bmatrix} -j \\ j \end{bmatrix}, \quad \underline{e}_{12} = \pm \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

so the 2D-RSSE performance shall not be dominated by the parallel transitions (i.e. error events of length 1) in the search of the subset trellis.

A close examination of the subset trellis reveals that

$$d'^2_{min} = d^2_{min} = 1.758$$

which is achieved by the 2-dimensional error events:

$$e_{min1,2} = \left\langle \pm \begin{bmatrix} 0 \\ j \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\rangle$$

$$e_{min3,4} = \left\langle \pm \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\rangle$$

$$e_{min5,6} = \left\langle \pm \begin{bmatrix} j \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\rangle$$

$$e_{min7,8} = \left\langle \pm \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\rangle$$

meaning that no severe degradation is expected in comparison to the full 16^8-state MLSE due to early merging error events. Still some degradation is expected due to the larger number of error events having the minimum distance dmin, resulting in Kdmin = 6, instead of Kdmin = 3 for the full MLSE.

Analysis and simulations show that the new scheme performs very closely to the full MLSE (having 16^8 states), and is superior to all other practical schemes by more than 2 dB at moderate to high SNR. While the space complexity of the 2D-RSSE is very low, due to the 16-

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state 2D-S trellis in use, it is of interest to estimate the computational complexity of the new scheme. The computational complexity in this case, might seem at first glance intimidating, due to the large number of branches leaving and entering each state (and hence branch metrics to be calculated). The number of branches here, following $D = 16$ and $M = 2$, is $16^2 = 256$. Notice that these branches are divided into $J\_1 = 2$ groups, each consisting of $256/2 = 128$ branches (or parallel transitions). A brute force method, may require at each iteration and for every state of the trellis, the computation of up to 256 branch metrics, followed by selecting the smallest metric from each group of 128 parallel transitions.

In order to reduce the number of calculations needed at each iteration, some embodiments of the invention make use of the methods to reduce complexity of each iteration, with adaptations to this special case. In some embodiments of the invention, the following algorithm, which is based on the decomposition of the branch metric computation is used:

$$
\begin{aligned}
\mathcal{B}_{\tau_{i,j}} &= \min_{\underline{x}_n \in \mathcal{R}_j(1)} \left\| \underline{r}_n - (\hat{p}_n(t_n), F) - F_0 \underline{x}_n \right\|^2 \\
&= \min_{\underline{x}_n \in \mathcal{R}_j(1)} \left\{ \left\| r_n^{(1)} - (\hat{p}_n(t_n), F)^{(1)} - [f_0 \ f_1][x_n^{(1)} \ x_n^{(2)}]^T \right\|^2 \right. \\
&\quad + \left. \left\| r_n^{(2)} - (\hat{p}_n(t_n), F)^{(2)} - [0 \ f_0][x_n^{(1)} \ x_n^{(2)}]^T \right\|^2 \right\} \\
&= \min_{\underline{x}_n \in \mathcal{R}_j(1)} \left\{ \left\| r_n^{(1)} - (\hat{p}_n(t_n), F)^{(1)} - f_0 x_n^{(1)} - f_1 x_n^{(2)} \right\|^2 \right. \\
&\quad + \left. \left\| r_n^{(2)} - (\hat{p}_n(t_n), F)^{(2)} - f_0 x_n^{(2)} \right\|^2 \right\}
\end{aligned}
$$

where

$$
\underline{x}_n = \begin{bmatrix} x_n^{(1)} \\ x_n^{(2)} \end{bmatrix}, \quad \underline{r}_n = \begin{bmatrix} r_n^{(1)} \\ r_n^{(2)} \end{bmatrix}, \quad (\hat{p}_n(t_n), F) = \begin{bmatrix} (\hat{p}_n(t_n), F)^{(1)} \\ (\hat{p}_n(t_n), F)^{(2)} \end{bmatrix}
$$

and

$$
F_0 = \begin{bmatrix} f_0 & f_1 \\ 0 & f_0 \end{bmatrix} = \begin{bmatrix} 1 & 0.05 \\ 0 & 1 \end{bmatrix}
$$

$Rj(1)$ for $j = 1,2$ are the two 4D families of the table, and $(\ )T$ stands for the transpose operation.

This leads to the following algorithm:

(1) For every state in the trellis, compute

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$$r_n^{(2)} - (\check{p}_n(t_n), F)^{(2)},$$

and find the closest point $X^{(2)}_n$ in each 2D subset (i.e. subsets {a, b, c, d}, following figure 8) and its metric. This may be done using simple slicing operations. Altogether it involves 4 slicing operations and 4 2D branch metric computations.

5　　(2) For the selected point in each 2D subset, $X^{(2)}_n$, compute

$$r_n^{(1)} - (\check{p}_n(t_n), F)^{(1)} - f_1 x_n^{(2)}$$

and find the closest point $X^{(1)}_n$ in each union of 2D subset pair (i.e. subsets {a U b} and {c U d}, following figure 8) and its metric. This follows the 2 4D families, as described in the table. Notice that when $X^{(2)}_n$ belongs to the set {a U b} than the closest point $X^{(1)}_n$ belonging to the

10　　set {a U b} corresponds to the 4D-family 1 in the table, whereas when $X^{(2)}_n$ belongs to the set {c U d} than the closest point $X^{(1)}_n$ in {a U b} corresponds to the 4D-family 2. Altogether, this step involves 4*2 = 8 slicing operations, and 8 2D branch metric computations.

(3) Compute the 8 branch metrics of the 4D points corresponding to family 1 (i.e. corresponding to the 4D types (a,aUb), (b,aUb), (c,cUd),(d,cUd)) and to family 2 (i.e.

15　　corresponding to the 4D types (a,cUd), (b,cUd), (c,aUb), (d,aUb)). This is done simply by adding the results attained from steps 1,2.

(4) Find the minimum branch metric, and the corresponding 4D symbol $\underline{X}_n$, for each 4D-family, out of each 4 branch metrics computed in the previous step. This involves 2*3 = 6 comparisons. In a particular embodiment the proposed algorithm reduces the number of

20　　required operations, for each iteration, and for every state of the trellis to:

12 slicing operations

12 computations of 2D branch metrics (plus 8 summations needed for the computation of the 4D branch metrics)

6 comparisons.

25　　This is generally better than the brute force method, which may require 256 4D branch metric computations and 254 comparisons. Therefore our 2D-RSSE is not only superior to other methods in performance, but also feasible to implement. In this example again the new scheme outperforms the existing ones in terms of performance/complexity tradeoff.

Alternatively to forming multi-dimensional symbols from consecutive transmitted

30　　symbols, in some embodiments of the invention, multi-dimensional input symbols are combined from M_1 consecutive or non consecutive input symbols, and multi-dimensional

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output samples are combined from M_2 consecutive or non consecutive input symbols. Optionally M_1 may be different than M_2. In That case, the following definitions may be used to get a MD-system model:

$$r_n = \sum_{k=L_1(n)}^{L_2(n)} F_k^{(n)} \underline{x}_{n-k} + \underline{w}_n$$

$$\underline{x}_n = \begin{bmatrix} x_{nM_1+a_{M_1-1}} \\ x_{nM_1+a_{M_1-2}} \\ \vdots \\ x_{nM_1+a_1} \\ x_{nM_1+a_0} \end{bmatrix}$$

5

$$\underline{r}_n = \begin{bmatrix} r_{nM_2+b_{M_2-1}} \\ r_{nM_2+b_{M_2-2}} \\ \vdots \\ r_{nM_2+b_1} \\ r_{nM_2+b_0} \end{bmatrix}$$

$F_k^{(n)}$ where $k = \{L_1(n), L_1(n)+1, L_1(n)+2, \dots L_2(n)-1, L_2(n)\}$ are $L_2(n) - L_1(n) + 1$ matrices of size $M_1 * M_2$ (having M_1 rows and M_2 columns) each, such that:

$$F_k^{(n)} = \begin{bmatrix} f_{n(M_2-M_1)+kM_1+b_{M_2-1}-a_{M_1-1}} & f_{n(M_2-M_1)+kM_1+b_{M_2-1}-a_{M_1-2}} & \cdots & f_{n(M_2-M_1)+kM_1+b_{M_2-1}-a_1} & f_{n(M_2-M_1)+kM_1+b_{M_2-1}-a_0} \\ f_{n(M_2-M_1)+kM_1+b_{M_2-2}-a_{M_1-1}} & f_{n(M_2-M_1)+kM_1+b_{M_2-2}-a_{M_1-2}} & \cdots & f_{n(M_2-M_1)+kM_1+b_{M_2-2}-a_1} & f_{n(M_2-M_1)+kM_1+b_{M_2-2}-a_0} \\ \vdots & \vdots & & \vdots \\ f_{n(M_2-M_1)+kM_1+b_1-a_{M_1-1}} & f_{n(M_2-M_1)+kM_1+b_1-a_{M_1-2}} & \cdots & f_{n(M_2-M_1)+kM_1+b_1-a_1} & f_{n(M_2-M_1)+kM_1+b_1-a_0} \\ f_{n(M_2-M_1)+kM_1+b_0-a_{M_1-1}} & f_{n(M_2-M_1)+kM_1+b_0-a_{M_1-2}} & \cdots & f_{n(M_2-M_1)+kM_1+b_0-a_1} & f_{n(M_2-M_1)+kM_1+b_0-a_0} \end{bmatrix}$$

where $L_1(n)$ and $L_2(n)$ are respectively the smallest and largest values of k for which $F^{(n)}_k$

10   differs from the zero matrix, and

$$\{a_0, a_1, \dots, a_{M_1-1}\} = \{0, 1, \dots, M_1 - 1\} \quad (mod \ M_1)$$

and

$$\{b_0, b_1, \dots, b_{M_2-1}\} = \{0, 1, \dots, M_2 - 1\} \quad (mod \ M_2)$$

Fig. 7 is a schematic block diagram of a multi-input multi-output (MIMO) symbol detector, known in the art. In some embodiment of this invention, Multi-Dimensional methods described above are used to recover the inputs of a multiple-input-multiple-output (MIMO) system, as shown for example in Fig. 7, based on observations that are made on the outputs of
5 the system. Such MIMO systems may comprise, for example digital communication, speech processing, and/or acoustics systems. The MIMO systems are meant herein to include single-input-multiple-output (SIMO) systems, such as occur, for example, in co-channel reception. In a M*M channel case, the relation between input and output may be defined as follows:

$$
\underline{r}_n = \sum_{k=0}^{L} F_k \underline{x}_{n-k} + \underline{w}_n
$$
$$
= \underline{y}_n + \underline{w}_n
$$

10 where $\underline{x}_n$ is the M-dimensional input sequence, consisting of elements from each of the M input sequences, such that:

$$
\underline{x}_n = \begin{bmatrix} x_{1n} \\ x_{2n} \\ \vdots \\ x_{3n} \\ x_{4n} \end{bmatrix}
$$

$\underline{r}_n$ is the M-dimensional output sequence, such that:

$$
\underline{r}_n = \begin{bmatrix} r_{1n} \\ r_{2n} \\ \vdots \\ r_{3n} \\ r_{4n} \end{bmatrix}
$$

15 F_k where k={0,1,2, ..., L} are L+1 matrices of size M*M each, such that:

$$
F_k = \begin{bmatrix} f_{11k} & f_{21k} & \cdots & f_{M1k} \\ f_{12k} & f_{22k} & \cdots & f_{M2k} \\ \vdots & \vdots & & \vdots \\ f_{1Mk} & f_{2Mk} & \cdots & f_{MMk} \end{bmatrix}
$$

$\underline{w}_n$ represents an M-dimensional noise sequence.

The multi-dimensional signal sequence:

$$\underline{y}_n \stackrel{\text{def}}{=} \sum_{k=0}^{L} F_k \underline{x}_{n-k}$$

is the multi-dimensional convolution of the multi-dimensional input sequence $\{ \underline{x}_n \}$ with the

5   finite impulse response $\{ F_k \}$.

The principles of the present invention may be applied also to systems with different numbers of input and output symbols. Furthermore, the principles of the present invention may be used with MD-symbols which represent a plurality of time sequences of multi-inputs and/or multi-outputs.

10   In some embodiments of the invention, the above described method may be used for combined detection and decoding. In an exemplary embodiment of the invention, a MD approach is used for system modeling for the channel or the code sequence or for both, followed by reducing the number of states and distinct transitions in the MD-trellis. Optionally a MD-subset trellis is generated such that only branches which are legal in accordance with the

15   coding scheme used by transmitter 22 remain in the trellis. Optionally a decoder and detector for MD-trellis coded modulations known in the art and ISI may be constructed using the above described embodiments.

The above described embodiments may be used in substantially any receiver, including receivers of data, voice, image and/or any other data. Furthermore, the above described

20   embodiments may be used with receivers which receive signals from substantially any medium including, wireless mediums (e.g., cellular, radio, satellite) and wired mediums (e.g., cables, fiber optics, telephone lines), as well as acoustic communication systems. Furthermore, the present invention may be used with point to point or point to multi-point systems.

In an exemplary application, a cellular telephone, for example using an FDMA,

25   TDMA, GSM or CDMA communication protocol implements the above equalizer methods, for example, as hardwired circuitry, such as an ASIC, or using firmware or software programmable circuits. Other exemplary receivers which may incorporate the above equalizer appartus (e.g. hardware and/or software designed and/or programmed to implement the equalizer methods), include, BlueTooth, cellular base stations, Modems, media readers such as

30   disk drives and optical media, and cable TV systems.

In an exemplary embodiment of the invention, a single equalizer can utilize multiple equalization methods, for example, one or more of the above described methods and, optionally, one or more multiple prior art methods. Optionally, the equalizer selects the method based on the noise levels, other noise characteristics, received signal quality and/or
5   available computational resources. Optionally, the equalizer includes a table that compares the computational requirements and/or signal detection qualities of the plurality of equalization methods.

In an exemplary embodiment of the invention, one or more of the above MD-RSSE methods is used to select optimal transmission sequences, instead of or in addition to being
10   used for extracting received signals, for example at cellular base stations, or for wireless transmission from cellular telephones. It is noted that the optimal sequence may change, for example, as a function of instantaneous interference characteristics.

In addition, the present invention may be used with communication channels of substantially any interference and/or noise patterns. In an exemplary embodiment of the
15   invention, the methods are used in applications where multiple reflections and transmission paths are expected, such as city networks and home networks. It is noted that although the system models described above are used in performing some of the acts of the methods of the present invention, these methods may be used even when the models or inaccurate or incorrect. Specifically, the noise and/or interference of the model may result from the transmitter, the
20   channel, the receiver and/or any other elements of the system. Furthermore, the noise and/or interference may result from other systems or may be generated intentionally. The present invention may be used with analog and/or digital systems. The present invention may also be used in systems other than data transmission systems, for example data recovery systems, such as magnetic recording systems.

25   It is further noted that the above equations may represent real and/or complex values. Likewise, the system and/or elements thereof may be complex or real.

It will be appreciated that the above described methods may be varied in many ways, including, changing the order of steps, and the exact implementation used. It should also be appreciated that the above described description of methods and apparatus are to be
30   interpreted as including apparatus and software for carrying out the methods and methods of using the apparatus.

The present invention has been described using non-limiting detailed descriptions of embodiments thereof that are provided by way of example and are not intended to limit the scope of the invention. It should be understood that features and/or steps described with respect to one embodiment may be used with other embodiments and that not all embodiments of the invention have all of the features and/or steps shown in a particular figure or described with respect to one of the embodiments. Variations of embodiments described will occur to persons of the art.

It is noted that some of the above described embodiments describe the best mode contemplated by the inventors and therefore include structure, acts or details of structures and acts that may not be essential to the invention and which are described as examples. Structure and acts described herein are replaceable by equivalents which perform the same function, even if the structure or acts are different, as known in the art. Therefore, the scope of the invention is limited only by the elements and limitations as used in the claims. When used in the following claims, the terms "comprise", "include", "have" and their conjugates mean "including but not limited to".

CLAIMS

1. A method of detecting symbols in a signal transfer system, comprising:

transferring symbols on a channel which may distort the symbols;

5   representing the transferred symbols by multi-dimensional (MD) symbols, which each includes a plurality of transferred symbols;

providing a trellis which has states which are defined by one or more MD symbols, and which has branches which correspond to respective MD symbols;

receiving a plurality of samples; and

10   calculating estimate values of one or more transferred symbols based on the received samples and the provided trellis.

2. A method according to claim 1, wherein the MD symbols comprise a plurality of transferred symbols which take on values from identical value sets.

15

3. A method according to claim 1 or claim 2, wherein at least one of the states of the provided trellis comprises a subset state which represents a plurality of different MD-symbol values.

20   4. A method according to claim 3, wherein the trellis comprises states which represent values of a predetermined number of MD symbols and wherein providing the trellis comprises generating the trellis by partitioning the possible values of each of the MD symbols in the states of the trellis into one or more subsets and wherein the states of the trellis comprise all the permutations of the subsets of the values of the multi-dimensional symbols.

25

5. A method according to claim 4, wherein partitioning the possible values of each of the MD symbols in the states of the trellis into one or more subsets comprises grouping the possible values of different MD symbols of the trellis states into different numbers of subsets.

30   6. A method according to claim 4, wherein partitioning the possible values of each of the MD symbols in the states of the trellis into one or more subsets comprises partitioning the possible values of the MD symbols which include earlier transmitted symbols into no more

subsets than the possible values of the MD symbols which were transmitted later were partitioned into.

7.   A method according to claim 4, wherein partitioning the possible values of each of the MD symbols in the states of the trellis into one or more subsets comprises partitioning the possible values of later transmitted symbols into subsets of which all the members are included in a single subset of earlier transmitted symbols.

8.   A method according to any of the preceding claims, comprising generating a model of the channel and wherein providing the trellis comprises providing a trellis generated responsive to the channel model.

9.   A method according to any of the preceding claims, wherein providing the trellis comprises providing a predetermined trellis.

10.   A method according to any of claims 1-8, wherein providing the trellis comprises providing a trellis determined responsive to the received samples.

11.   A method according to any of claims 1-8, wherein providing the trellis comprises optimizing a trellis for a particular transmission situation.

12.   A method according to any of claims 1-8, wherein providing the trellis comprises:
       generating a plurality of possible trellises;
       determining for each of the possible trellises one or more of a performance or a complexity parameter; and
       selecting one of said trellises responsive to said determining.

13.   An equalizer, comprising:
       a input channel for receiving received signals;
       a processor for applying a MD trellis methods in accordance with any of the preceding claims; and

40

an output channel for providing an estimate of symbols received on said input channel, based on said applying.

14. A cellular telephone comprising an equalizer according to claim 13.
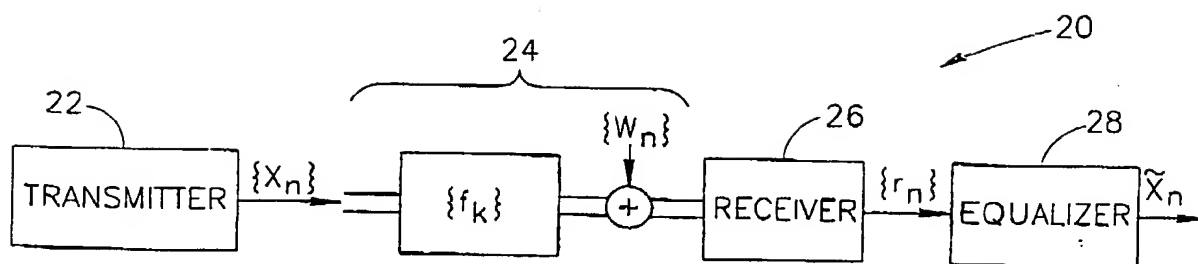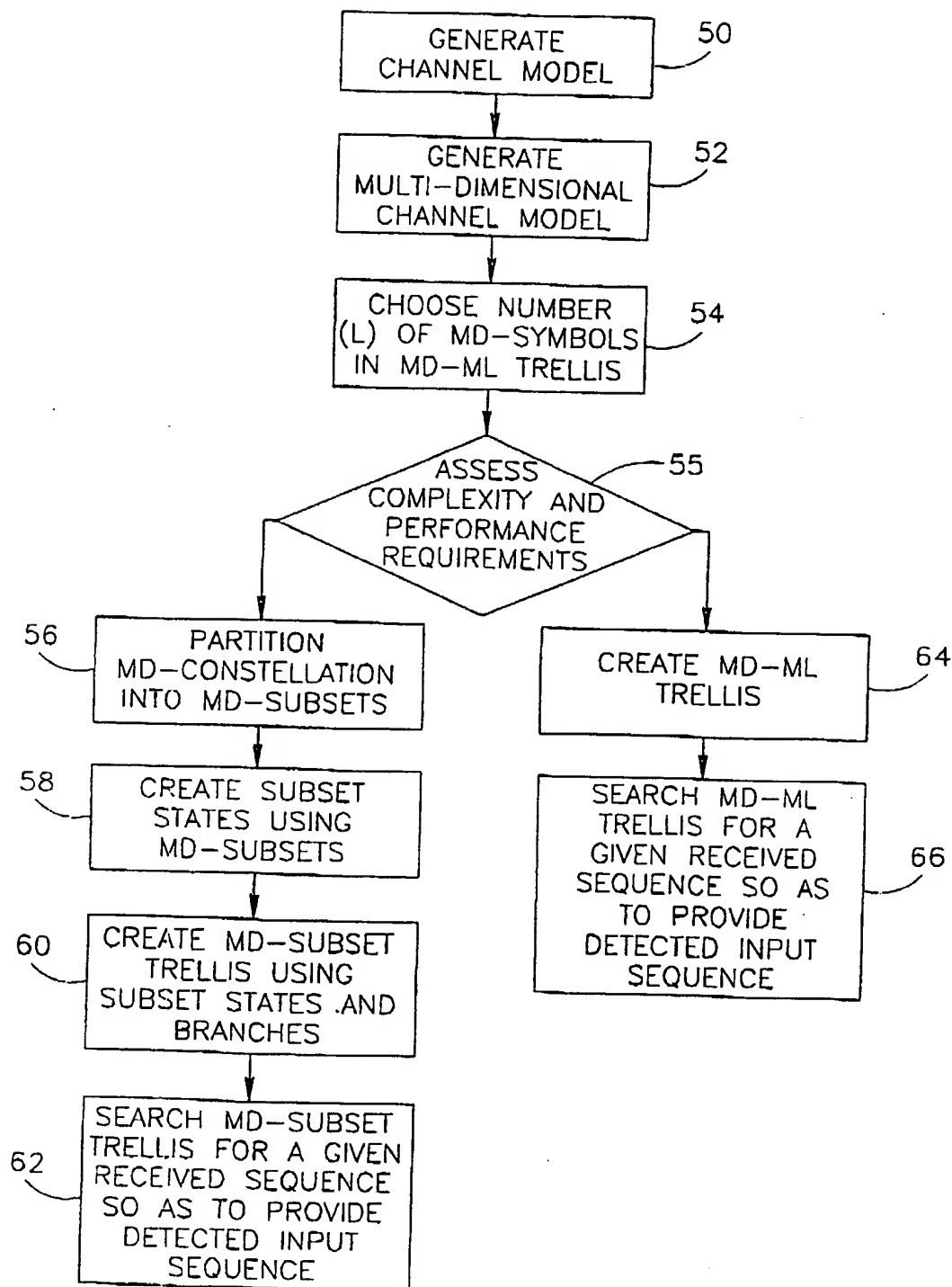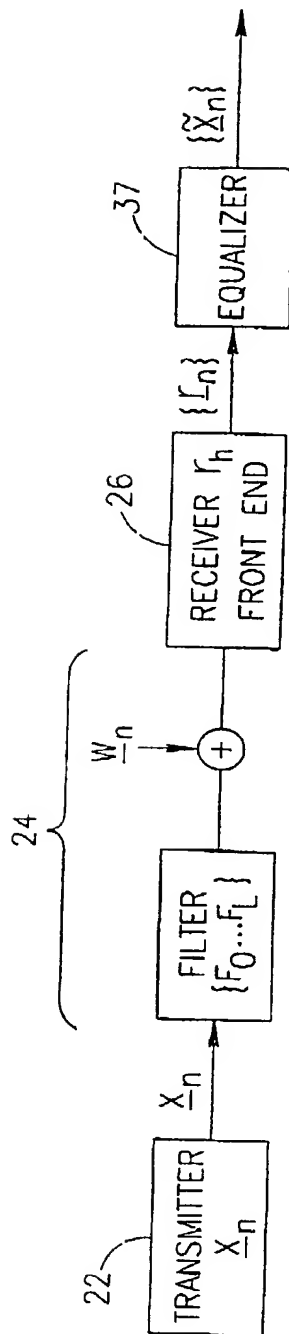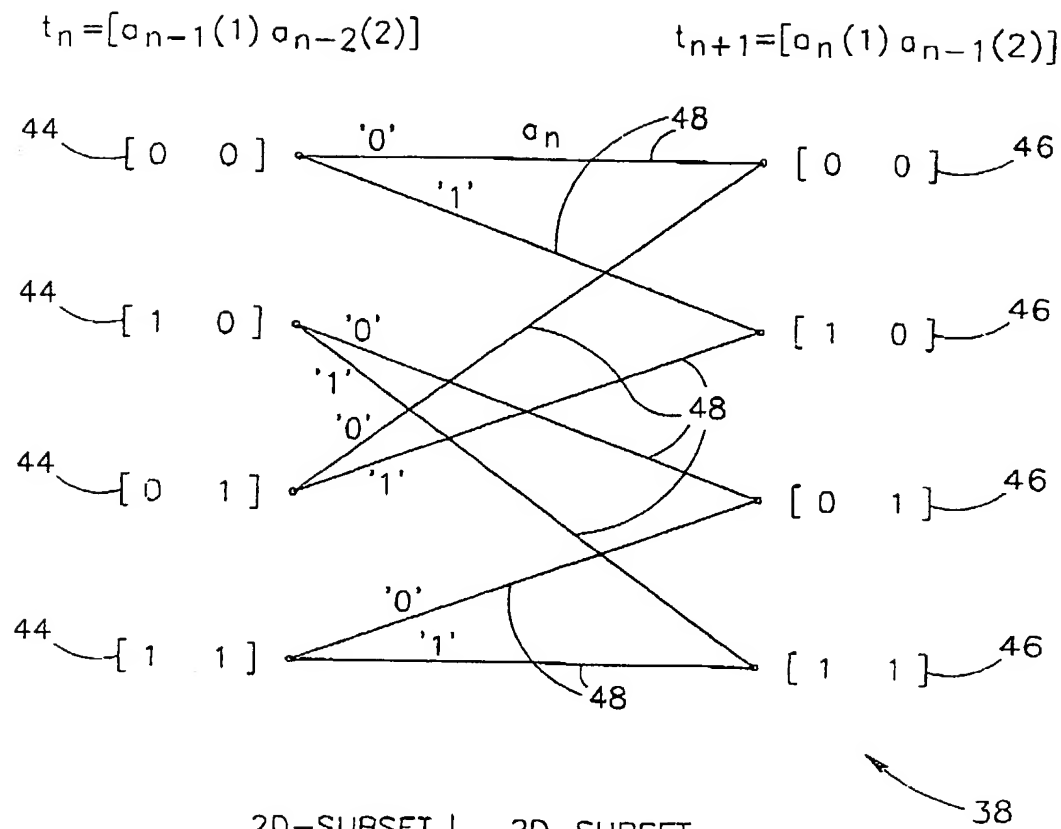
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FIG.1

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```
┌──────────────────────┐
│      GENERATE         │── 50
│   CHANNEL MODEL       │
└──────────────────────┘
           │
           ▼
┌──────────────────────┐
│      GENERATE         │── 52
│  MULTI-DIMENSIONAL    │
│   CHANNEL MODEL       │
└──────────────────────┘
           │
           ▼
┌──────────────────────┐
│   CHOOSE NUMBER       │── 54
│ (L) OF MD-SYMBOLS     │
│  IN MD-ML TRELLIS     │
└──────────────────────┘
```

FIG.2

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FIG.3

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$$t_n = [a_{n-1}(1) \ a_{n-2}(2)] \qquad\qquad t_{n+1} = [a_n(1) \ a_{n-1}(2)]$$

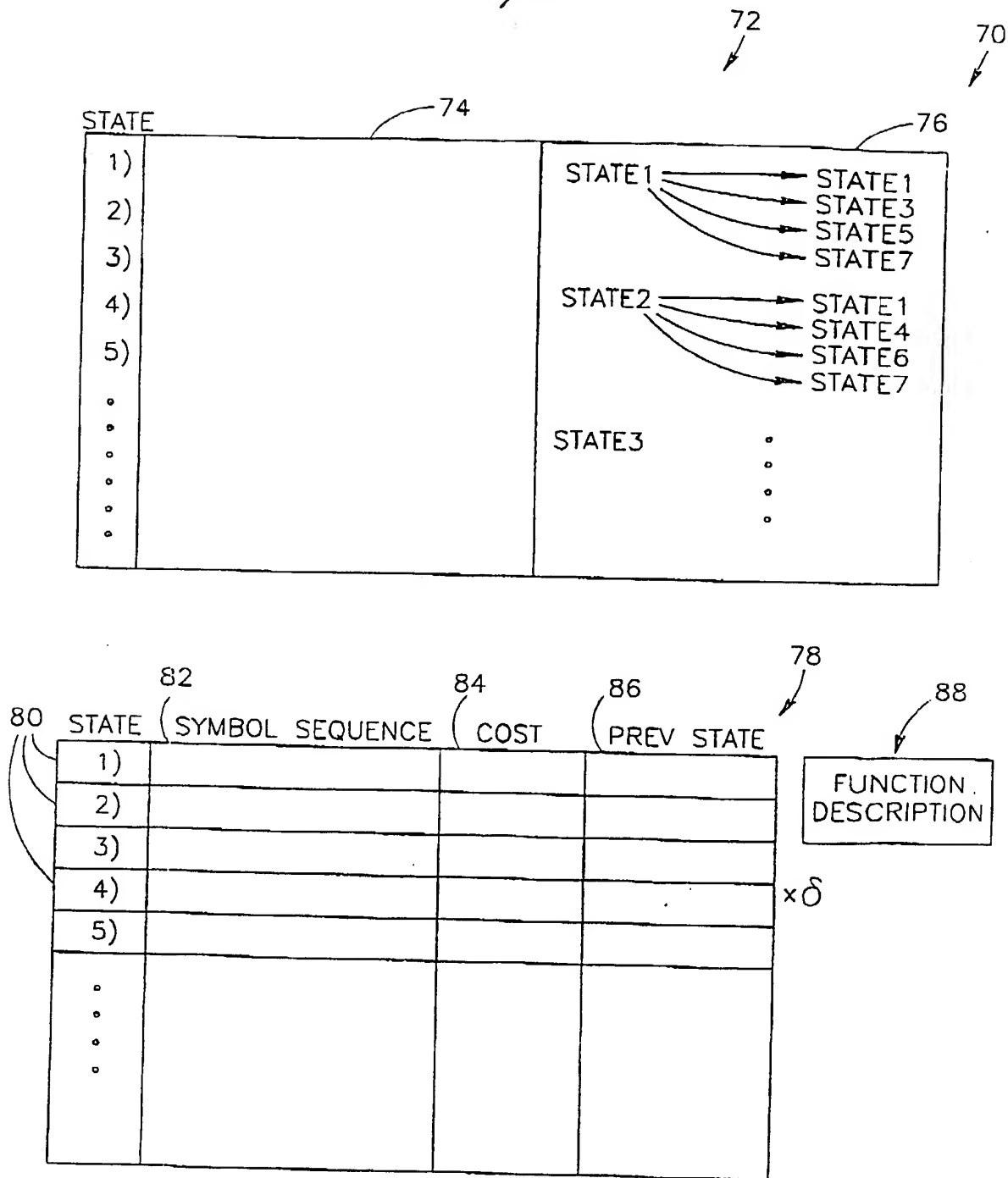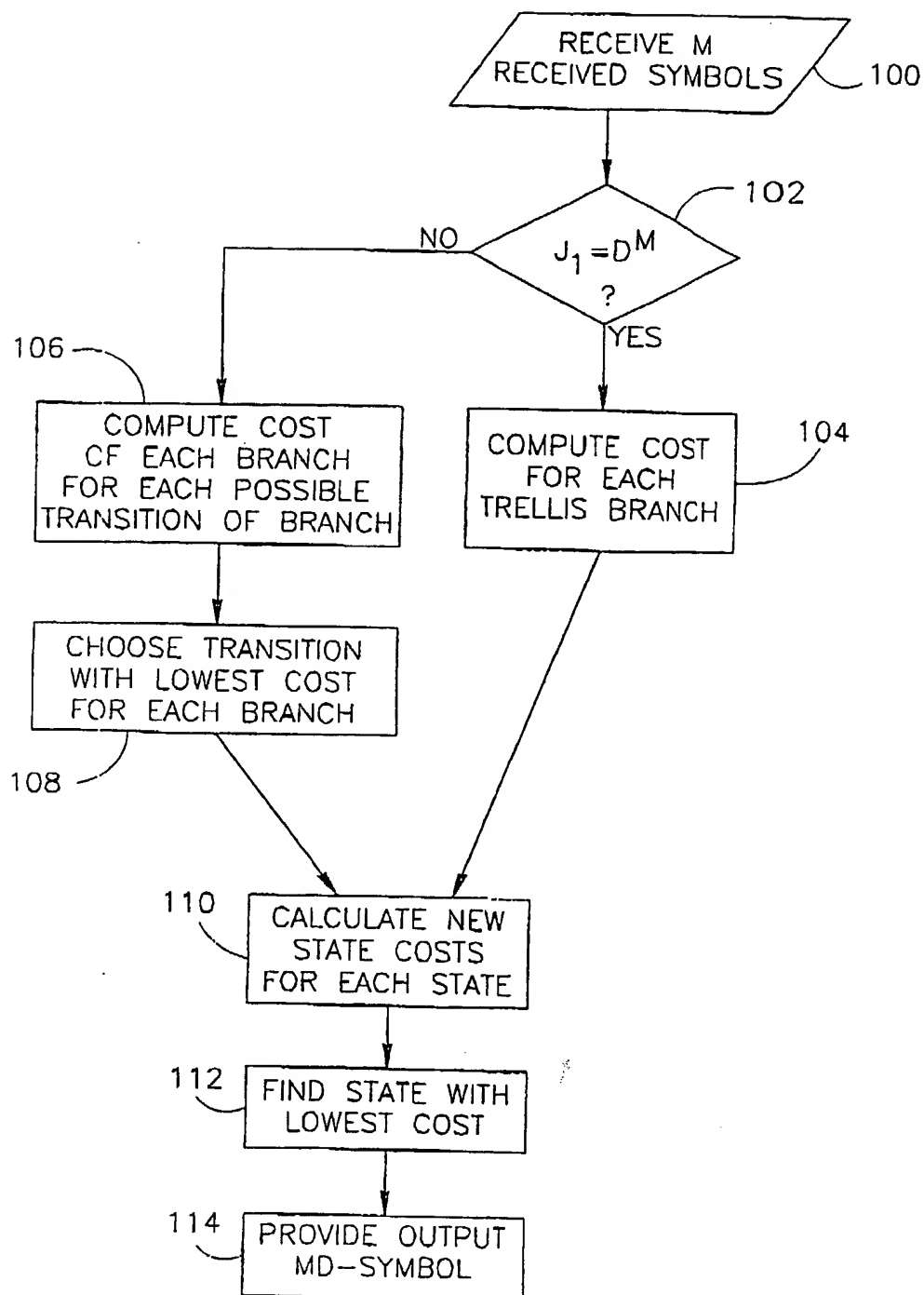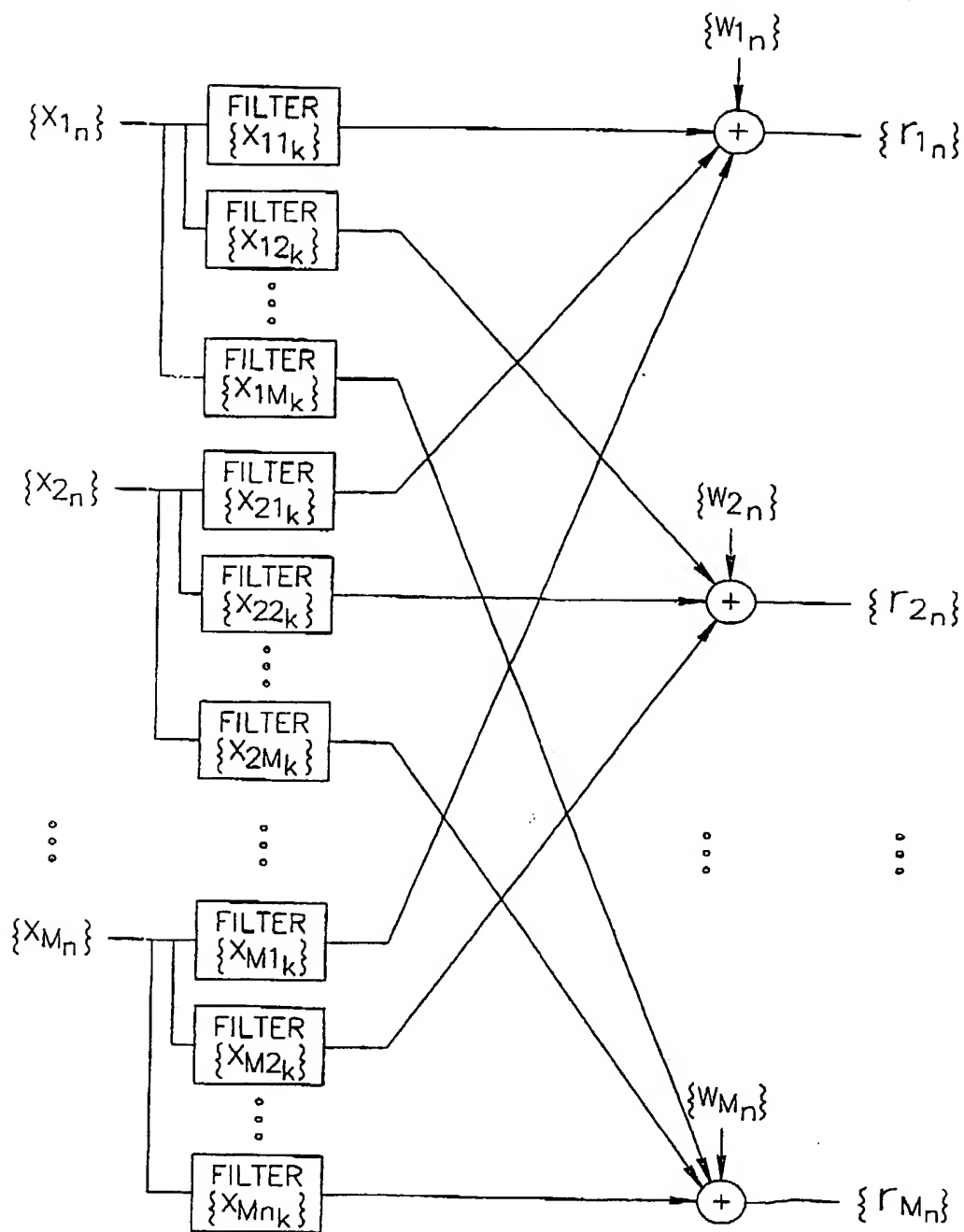| 2D-SUBSET INDEX	2D-SUBSET ELEMENTS
'0'	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$
'1'	$\begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

FIG.4

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STATE

1)	STATE1 ──→ STATE1
2)	STATE3
3)	STATE5
4)	STATE7
5)	STATE2 ──→ STATE1

FIG.5

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RECEIVE M
RECEIVED SYMBOLS   100

$J_1 = D^M$?   102

NO

YES

COMPUTE COST
CF EACH BRANCH
FOR EACH POSSIBLE
TRANSITION OF BRANCH   106

COMPUTE COST
FOR EACH
TRELLIS BRANCH   104

CHOOSE TRANSITION
WITH LOWEST COST
FOR EACH BRANCH

108

CALCULATE NEW
STATE COSTS
FOR EACH STATE   110

FIND STATE WITH
LOWEST COST   112

PROVIDE OUTPUT
MD-SYMBOL   114

FIG.6

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FIG.7

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FIG. 8

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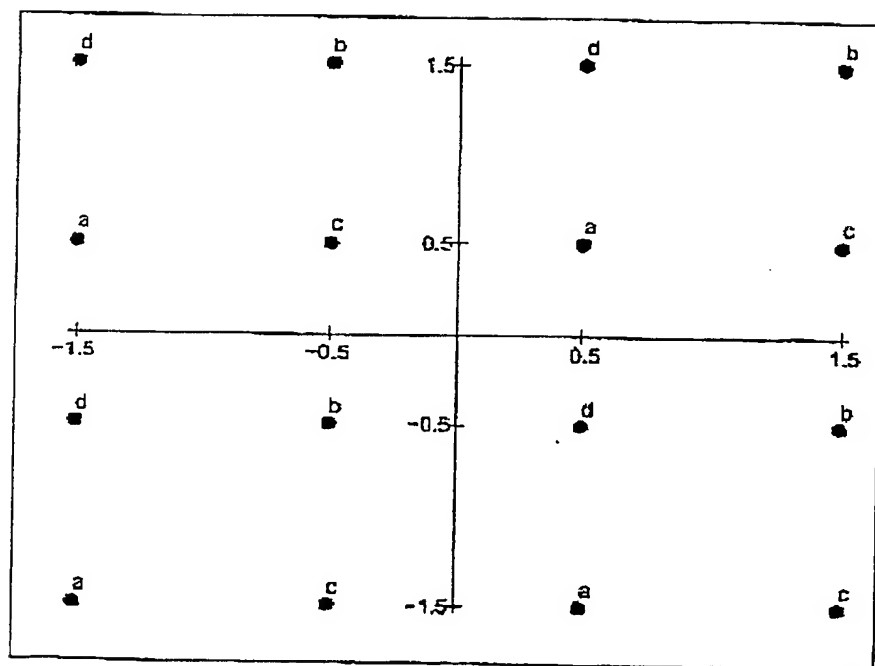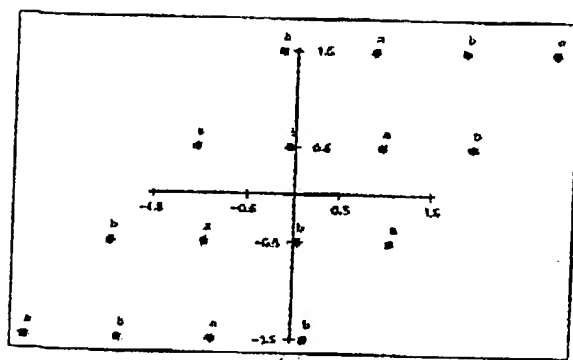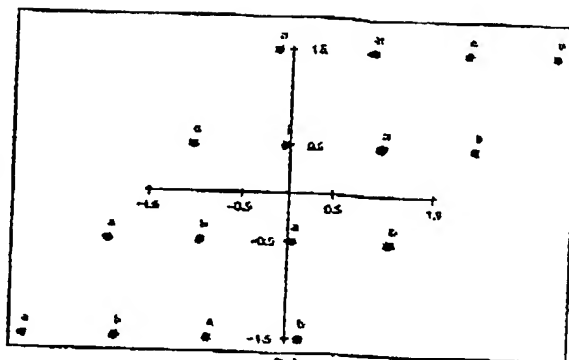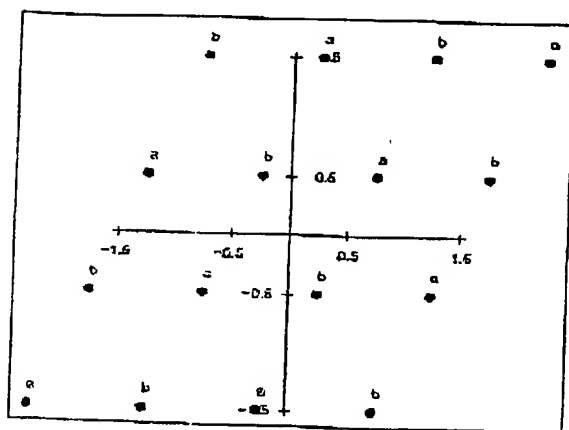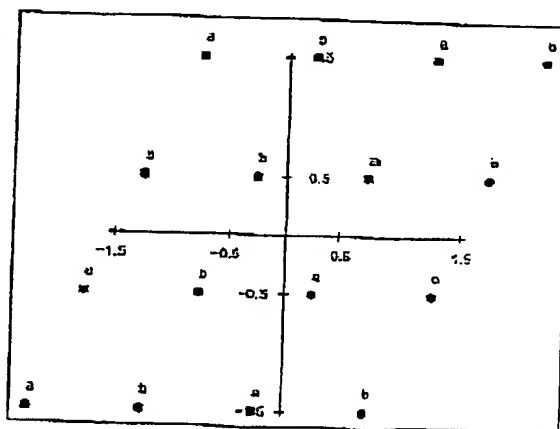
FIG. 9

Inte   .onal Application No

PCT/US 00/28536

A. CLASSIFICATION OF SUBJECT MATTER
IPC 7   H04L25/03

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC 7   H04L

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

EPO-Internal, WPI Data, PAJ, INSPEC

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	VEDAT EYUBOGLU M ET AL: "ADVANCED MODULATION TECHNIQUES FOR V.FAST" EUROPEAN TRANSACTIONS ON TELECOMMUNICATIONS AND RELATED TECHNOLOGIES,AEI, MILANO,IT, vol. 4, no. 3, 1 May 1993 (1993-05-01), pages 243-256, XP000385751 ISSN: 1120-3862 the whole document	1-14
X	EP 0 536 948 A (AMERICAN TELEPHONE & TELEGRAPH) 14 April 1993 (1993-04-14) abstract column 2, line 26 -column 3, line 5 figure 1	1-14
	-/--	

[X] Further documents are listed in the continuation of box C. [X] Patent family members are listed in annex.

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Date of the actual completion of the international search	Date of mailing of the international search report
20 February 2001	27/02/2001

Name and mailing address of the ISA	Authorized officer
European Patent Office, P.B. 5818 Patentlaan 2 NL - 2280 HV Rijswijk Tel. (+31-70) 340-2040, Tx. 31 651 epo nl, Fax: (+31-70) 340-3016	Toumpoulidis, T

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C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication,where appropriate, of the relevant passages	Relevant to claim No.
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